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Ghost Imaging: An Overview

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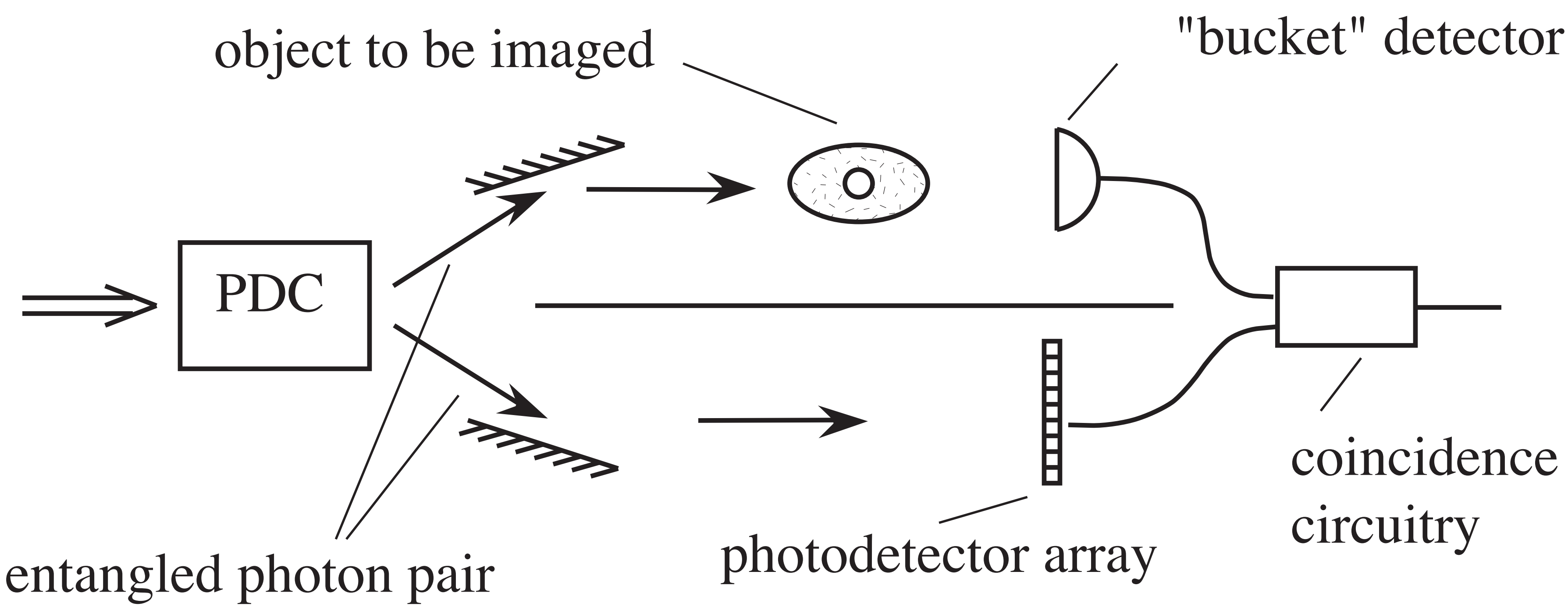
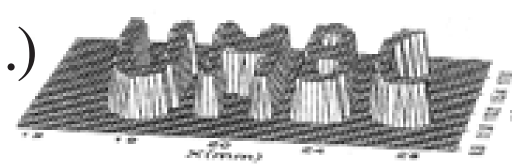
With special thanks to Kam Wai Clifford Chan, Anand Jha, Mehul Malik, Colin O'Sullivan, Heedeuk Shin, and Petros Zerom

Ghost (Coincidence) Imaging



- Obvious applicability to remote sensing!
  (imaging under adverse situations, bio, two-color, etc.)
- Is this a purely quantum mechanical process? (No)
- Can Brown-Twiss intensity correlations lead to ghost imaging? (Yes)

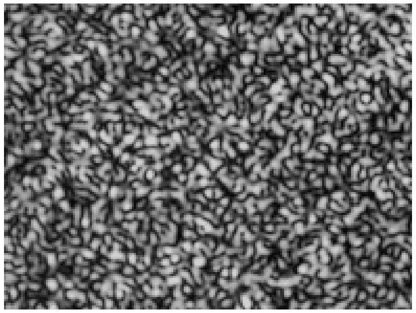


Strekalov et al., Phys. Rev. Lett. 74, 3600 (1995).
Pittman et al., Phys. Rev. A 52 R3429 (1995).
Abouraddy et al., Phys. Rev. Lett. 87, 123602 (2001).
Bennink, Bentley, and Boyd, Phys. Rev. Lett. 89 113601 (2002).
Bennink, Bentley, Boyd, and Howell, PRL 92 033601 (2004)
Gatti, Brambilla, and Lugiato, PRL 90 133603 (2003)
Gatti, Brambilla, Bache, and Lugiato, PRL 93 093602 (2003)

Thermal Ghost Imaging

Instead of using quantum-entangled photons, one can perform ghost imaging using the correlations of a thermal light source, as predicted by Gatti et al. 2004.

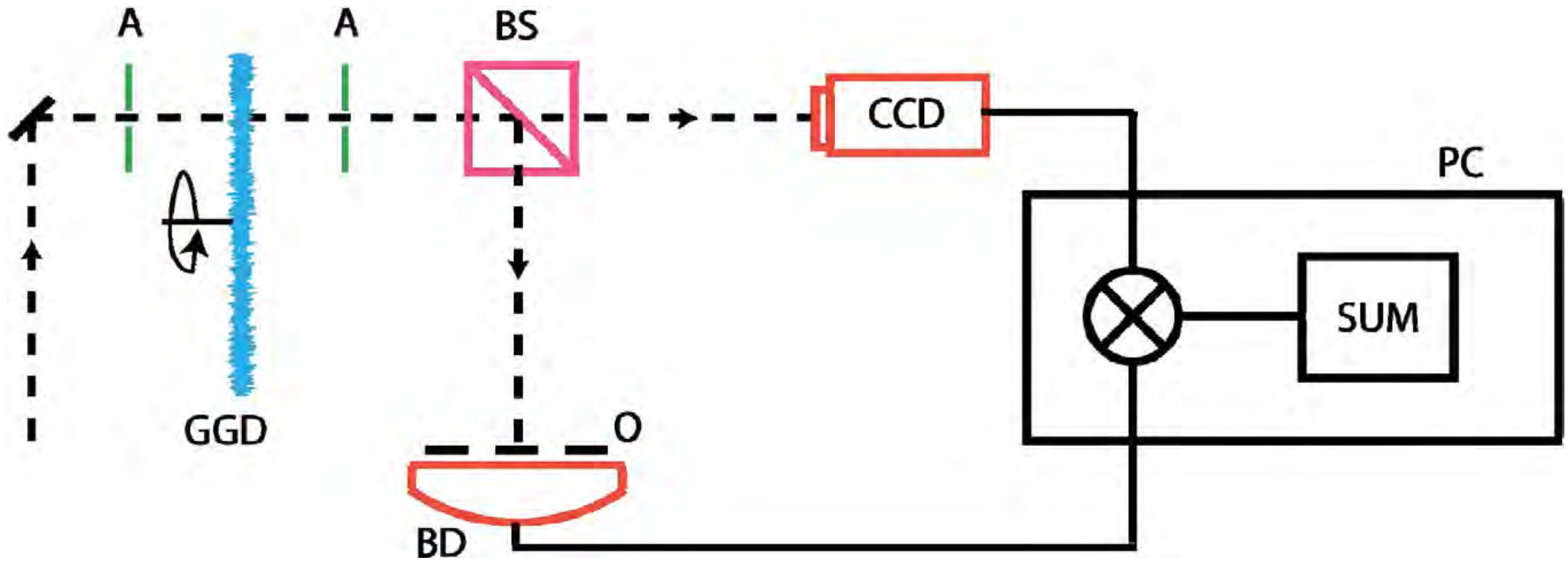
Recall that the intensity distribution of thermal light looks like a speckle pattern.



We use pseudothermal light in our studies: we create a speckle pattern with the same statistical properties as thermal light by scattering a laser beam off a rotating ground glass plate.

Thermal ghost imaging has been observed previously by several groups; our interest is in performing careful studies of its properties.

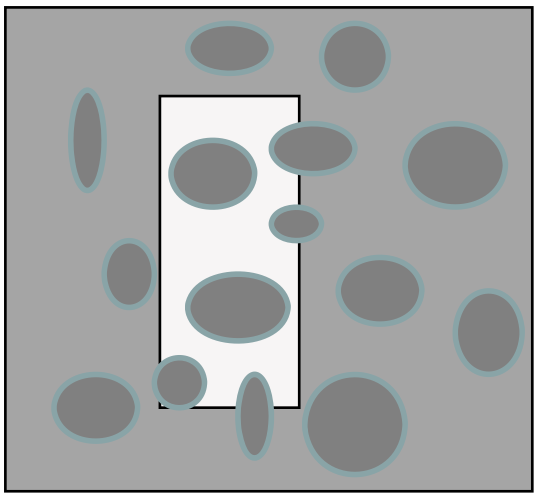
How does thermal ghost imaging work?



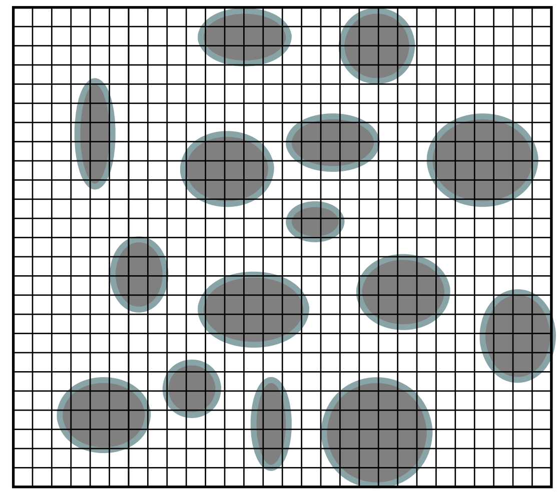
- Ground glass disk (GGD) and beam splitter (BS) create two identical speckle patterns
- Many speckles are blocked by the opaque part of object, but some are transmitted, and their intensities are summed by BD
- CCD camera measures intensity distribution of speckle pattern
- Each speckle pattern is multiplied by the output of the BD
- Results are averaged over a large number of frames.

Origin of Thermal Ghost Imaging

Create identical speckle patterns in each arm.



object arm
(bucket detector)



reference arm
(pixelated imaging detector)

$g_1(x,y)$ = (total transmitted power) x (intensity at each point x,y)

Average over many speckle patterns

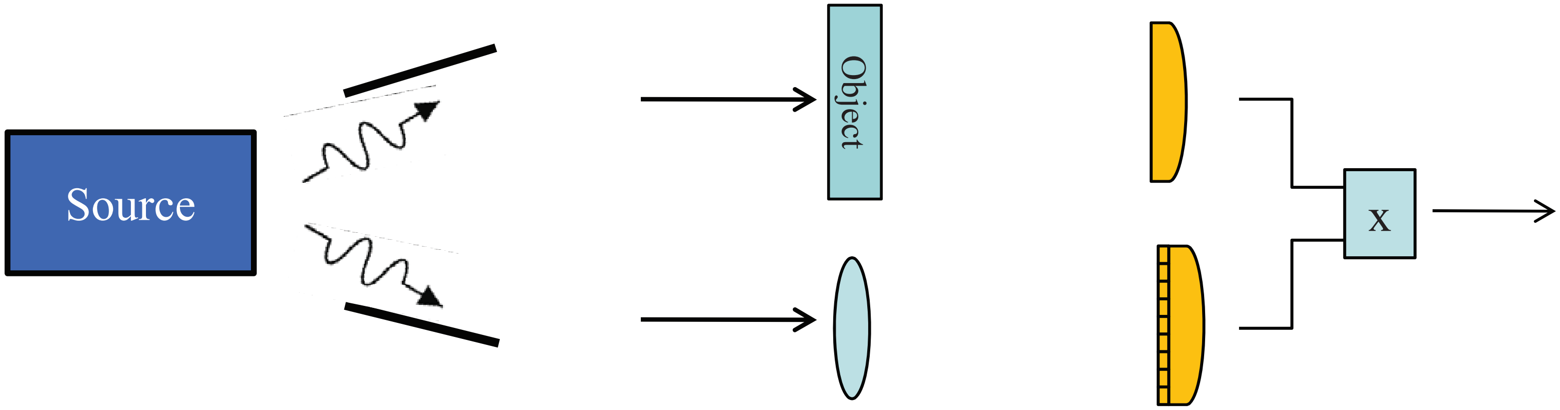
Resolution, Contrast, and Noise of Quantum and Thermal Ghost Images

Malcolm O'Sullivan, Mehul Malik, Kam Wai Clifford Chan, and Robert Boyd

Institute of Optics, University of Rochester, Rochester, NY USA

IQEC, 6:00 p.m. June 4th, 2009

Quantum and Thermal Ghost Imaging



Images formed in the correlation signal between light detected at the bucket detector and CCD.

Typical Requirements:

- The source needs to produce correlated fields between object and CCD planes. Correlations can either be quantum (e.g. SPDC) or classical (e.g. “thermal”) .
- Quantum – single photon detection with coincidence circuitry.
- Thermal – measure intensity correlations (can operate at high light levels).

Which is better? And under what circumstances?

Quantum and Thermal Ghost Imaging

Several papers addressing differences:

- Chan et al., PRA **79**, 033808 (2009)
- Erkmen and Shapiro, PRA **79**, 023833 (2009) & PRA **78**, 023835 (2008)
- D'Angelo et al., PRA **72**, 013810 (2005)
- Gatti et al., PRL **93**, 093602 (2004)

Quantum Ghost Imaging

- Uses entangled photon pairs from SPDC
- Weak source of light (~10^6 photons/s)
- Coincident photon detection events recorded to form image
- Resolution limited by phase-matching in NL crystal
- No background signal
- Photon-counting noise

Thermal Ghost Imaging

- Uses two-copies of light with limited spatial coherence ("thermal")
- Source of light can be very intense
- Images formed in correlation signal
- Resolution limited by coherence area
- Large background signal
- Noise due to thermal-statistics of the light

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Nonlinear Optics Group

Questions

- How do the images produced by the two methods differ under realistic experimental conditions?
  - Resolution
  - Contrast
  - Signal to Noise Ratio
- What are the scaling laws associated with the contrast and signal to noise ratio of the images?

Model System

Parameters:

K – number of realizations

w – transverse size of source

$A = \pi w^2$

σ – transverse correlation length at source plane

Δ – area of one pixel on CCD

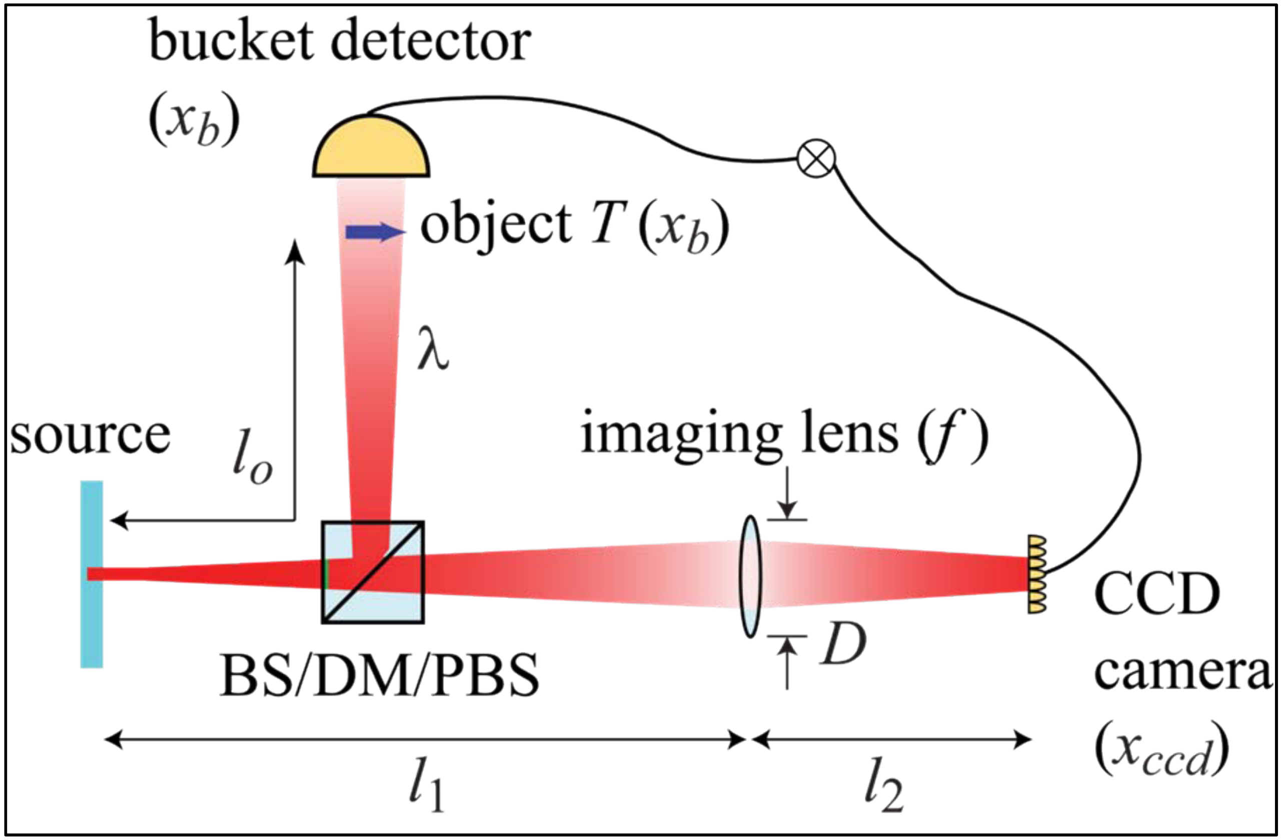


Image signal operator at pixel $\mathbf{x}_j$

$$\hat{S}(\mathbf{x}_k) = \sum_{k}^{K} \int_{\Delta} d\mathbf{x}_{\rm ccd} \int d\mathbf{x}_{\rm b}\ \hat{a}^{\dagger}_{k,\rm b}(\mathbf{x}_{\rm b})\hat{a}^{\dagger}_{k,\rm ccd}(\mathbf{x}_{\rm ccd})\hat{a}_{k,\rm b}(\mathbf{x}_{\rm b})\hat{a}_{k,\rm ccd}(\mathbf{x}_{\rm ccd})\, T(\mathbf{x}_{\rm b})$$

Formalism

Describe source by two-photon wavefunction or classical coherence function

$$|\psi\rangle = \frac{2}{(\pi w \sigma)^2} \int d\mathbf{x}_\mathrm{i} \int d\mathbf{x}_\mathrm{s}\, e^{-\frac{|\mathbf{x}_\mathrm{i}+\mathbf{x}_\mathrm{s}|^2}{4w^2}} e^{-\frac{|\mathbf{x}_\mathrm{i}-\mathbf{x}_\mathrm{s}|^2}{2\sigma^2}}\; \hat{a}_\mathrm{s}^\dagger(\mathbf{x}_\mathrm{s})\hat{a}_\mathrm{i}^\dagger(\mathbf{x}_\mathrm{i})|0,0\rangle \qquad \text{(Quantum)}$$

$$W(\mathbf{x}_1,\mathbf{x}_2) = \langle \hat{a}^\dagger(\mathbf{x}_1)\hat{a}(\mathbf{x}_2)\rangle = \frac{\overline{n}}{\pi w^2}\, e^{-\frac{|\mathbf{x}_1+\mathbf{x}_2|^2}{4w^2}} e^{-\frac{|\mathbf{x}_1-\mathbf{x}_2|^2}{2\sigma^2}} \qquad \text{(Thermal)}$$

$\overline{n}$ = total number of photons emitted by source (in speckle pattern)

The image signal (ghost image) is given by

$$\hat{S}(\mathbf{x}_j) = \sum_k^K \int_\Delta d\mathbf{x}_\mathrm{ccd} \int d\mathbf{x}_\mathrm{b}\; \hat{a}_{k,\mathrm{b}}^\dagger(\mathbf{x}_\mathrm{b})\hat{a}_{k,\mathrm{ccd}}^\dagger(\mathbf{x}_\mathrm{ccd})\hat{a}_{k,\mathrm{b}}(\mathbf{x}_\mathrm{b})\hat{a}_{k,\mathrm{ccd}}(\mathbf{x}_\mathrm{ccd})\, T(\mathbf{x}_\mathrm{b})$$

And the mean image signal is given by:

$$\langle \hat{S}(\mathbf{x}_j)\rangle = K\Delta \int d\mathbf{x}_\mathrm{b}\; G^{(2)}(\mathbf{x}_\mathrm{b},\mathbf{x}_\mathrm{ccd}) T(\mathbf{x}_\mathrm{b})$$

Formalism

Describe source by two-photon wavefunction or classical coherence function

$$|\psi\rangle = \frac{2}{(\pi w \sigma)^2} \int d\mathbf{x}_\mathrm{i} \int d\mathbf{x}_\mathrm{s}\, e^{-\frac{|\mathbf{x}_\mathrm{i}+\mathbf{x}_\mathrm{s}|^2}{4w^2}} e^{-\frac{|\mathbf{x}_\mathrm{i}-\mathbf{x}_\mathrm{s}|^2}{2\sigma^2}}\, \hat{a}_\mathrm{s}^\dagger(\mathbf{x}_\mathrm{s})\hat{a}_\mathrm{i}^\dagger(\mathbf{x}_\mathrm{i})|0,0\rangle \qquad \text{(Quantum)}$$

$$W(\mathbf{x}_1,\mathbf{x}_2) = \langle \hat{a}^\dagger(\mathbf{x}_1)\hat{a}(\mathbf{x}_2)\rangle = \frac{\overline{n}}{\pi w^2}\, e^{-\frac{|\mathbf{x}_1+\mathbf{x}_2|^2}{4w^2}} e^{-\frac{|\mathbf{x}_1-\mathbf{x}_2|^2}{2\sigma^2}} \qquad \text{(Thermal)}$$

Mean image signal : $\langle \hat{S}(\mathbf{x}_j)\rangle = K\Delta \int d\mathbf{x}_\mathrm{b}\, G^{(2)}(\mathbf{x}_\mathrm{b},\mathbf{x}_\mathrm{ccd})T(\mathbf{x}_\mathrm{b})$

Use paraxial propagators to relate $\hat{a}_{\mathrm{b,ccd}}$ at detection planes to the source plane.

$$\hat{a}_\mathrm{b}(\mathbf{x}_\mathrm{b}) = \int d\mathbf{x}_\mathrm{i}\; h_\mathrm{b}(\mathbf{x}_\mathrm{i};\mathbf{x}_\mathrm{b})\hat{a}_\mathrm{i}(\mathbf{x}_\mathrm{i})$$

$$\hat{a}_\mathrm{ccd}(\mathbf{x}_\mathrm{ccd}) = \int d\mathbf{x}_\mathrm{s}\; h_\mathrm{ccd}(\mathbf{x}_\mathrm{s};\mathbf{x}_\mathrm{ccd})\hat{a}_\mathrm{s}(\mathbf{x}_\mathrm{s})$$

Formalism

Describe source by two-photon wavefunction or classical coherence function

$$|\psi\rangle = \frac{2}{(\pi w \sigma)^2} \int d\mathbf{x}_\mathrm{i} \int d\mathbf{x}_\mathrm{s}\, e^{-\frac{|\mathbf{x}_\mathrm{i}+\mathbf{x}_\mathrm{s}|^2}{4w^2}} e^{-\frac{|\mathbf{x}_\mathrm{i}-\mathbf{x}_\mathrm{s}|^2}{2\sigma^2}}\ \hat{a}_\mathrm{s}^\dagger(\mathbf{x}_\mathrm{s})\hat{a}_\mathrm{i}^\dagger(\mathbf{x}_\mathrm{i})|0,0\rangle \qquad \text{(Quantum)}$$

$$W(\mathbf{x}_1,\mathbf{x}_2) = \langle \hat{a}^\dagger(\mathbf{x}_1)\hat{a}(\mathbf{x}_2)\rangle = \frac{\overline{n}}{\pi w^2}\, e^{-\frac{|\mathbf{x}_1+\mathbf{x}_2|^2}{4w^2}} e^{-\frac{|\mathbf{x}_1-\mathbf{x}_2|^2}{2\sigma^2}} \qquad \text{(Thermal)}$$

Mean image signal : $\langle \hat{S}(\mathbf{x}_j)\rangle = K\Delta \int d\mathbf{x}_\mathrm{b}\, G^{(2)}(\mathbf{x}_\mathrm{b}, \mathbf{x}_\mathrm{ccd}) T(\mathbf{x}_\mathrm{b})$

Use paraxial propagators to relate $\hat{a}_{\mathrm{b,ccd}}$ at detection planes to the source plane.

$$\hat{a}_\mathrm{b}(\mathbf{x}_\mathrm{b}) = \frac{1}{\sqrt{2}}\left(i\ \hat{v}(\mathbf{x}_\mathrm{b}) + \int d\mathbf{x}\ h_\mathrm{b}(\mathbf{x};\mathbf{x}_\mathrm{b})\hat{a}(\mathbf{x})\right)$$

$$\hat{a}_\mathrm{ccd}(\mathbf{x}_\mathrm{ccd}) = \frac{1}{\sqrt{2}}\left(i\ \hat{v}(\mathbf{x}_\mathrm{ccd}) + \int d\mathbf{x}\ h_\mathrm{ccd}(\mathbf{x};\mathbf{x}_\mathrm{ccd})\hat{a}(\mathbf{x})\right)$$

in principle, include vacuum noise introduced by beam splitterv

Formalism

Describe source by two-photon wavefunction or classical coherence function

$$|\psi\rangle = \frac{2}{(\pi w \sigma)^2} \int d\mathbf{x}_{\rm i} \int d\mathbf{x}_{\rm s}\; e^{-\frac{|\mathbf{x}_{\rm i}+\mathbf{x}_{\rm s}|^2}{4w^2}} e^{-\frac{|\mathbf{x}_{\rm i}-\mathbf{x}_{\rm s}|^2}{2\sigma^2}}\; \hat{a}_{\rm s}^{\dagger}(\mathbf{x}_{\rm s})\hat{a}_{\rm i}^{\dagger}(\mathbf{x}_{\rm i})|0,0\rangle \qquad \text{(Quantum)}$$

$$W(\mathbf{x}_1,\mathbf{x}_2) = \langle \hat{a}^{\dagger}(\mathbf{x}_1)\hat{a}(\mathbf{x}_2)\rangle = \frac{\overline{n}}{\pi w^2}\, e^{-\frac{|\mathbf{x}_1+\mathbf{x}_2|^2}{4w^2}} e^{-\frac{|\mathbf{x}_1-\mathbf{x}_2|^2}{2\sigma^2}} \qquad \text{(Thermal)}$$

Mean image signal : $\langle \hat{S}(\mathbf{x}_j)\rangle = K\Delta \int d\mathbf{x}_{\rm b}\; G^{(2)}(\mathbf{x}_{\rm b},\mathbf{x}_{\rm ccd})T(\mathbf{x}_{\rm b})$

$$\langle \hat{S}(\mathbf{x}_j)\rangle_{\rm quantum} = \alpha K \Delta e^{-\mathbf{x}_j^2/\Delta^2_{\rm FOV,+}} \int d\mathbf{x}_{\rm b} \exp\left[-\frac{|\mathbf{x}_{\rm b}-\mathbf{x}_j/m_+|^2}{\Delta^2_{\rm PSF,+}}\right] T(\mathbf{x}_{\rm b})$$

$$\langle \hat{S}(\mathbf{x}_j)\rangle_{\rm thermal} = \beta K \left(\frac{\overline{n}}{2}\right)^2 \Delta e^{-\mathbf{x}_j^2/\Delta^2_{\rm FOV,-}} \int d\mathbf{x}_{\rm b} \left(1+\exp\left[-\frac{|\mathbf{x}_{\rm b}-\mathbf{x}_j/m_-|^2}{\Delta^2_{\rm PSF,-}}\right]\right) T(\mathbf{x}_{\rm b})$$

Resolution

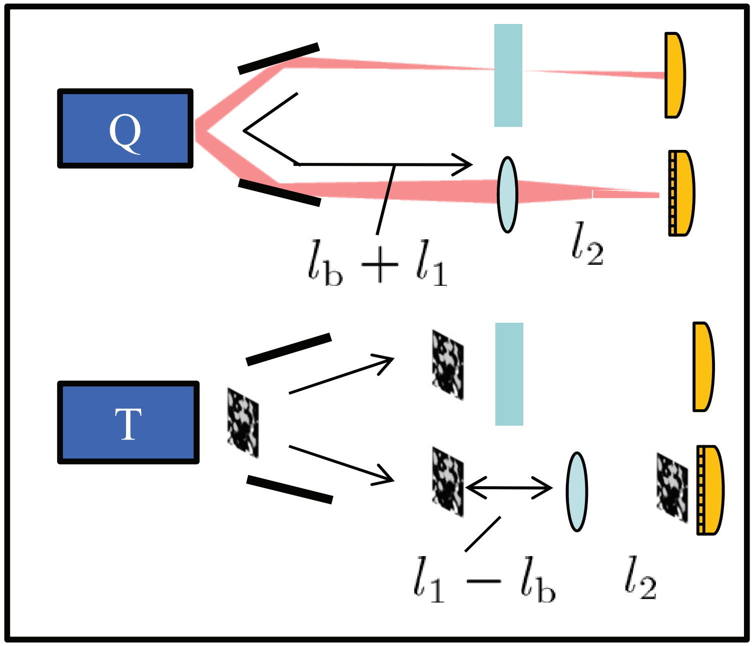
We find Δ_{PSF} is a minimum when:

+ quantum
− thermal

$$\frac{1}{l_1 \pm l_b} + \frac{1}{l_2} - \frac{1}{f} = 0$$

In this case, the magnification is given by

$$m = \frac{-l_2}{l_1 \pm l_b}$$



$$\Delta_{\mathrm{PSF}} = \sqrt{\sigma^2 + \frac{\lambda^2 (l_1 \pm l_b)^2}{(2\pi D)^2}} \quad = \sqrt{\sigma^2 + \left(\frac{\lambda}{2\pi}\right)^2 \left(\frac{1+|m|}{|m|} F\right)^2}$$

$$F \equiv f/D$$

$$\sigma = 0 \rightarrow \quad |m|\Delta_{\mathrm{PSF}} \propto \lambda(1+|m|)F$$

Contrast

- First calcuate the signal for one particular pixel
- Neglect diffraction and take limit of large source size *w* and small correlation size σ

$$\langle \hat{S}(\mathbf{x}_j) \rangle_{\text{quantum}} = \alpha K \Delta e^{-\mathbf{x}_j^2/\Delta^2_{\text{FOV},+}} \int d\mathbf{x}_{\text{b}} \exp\left[-\frac{|\mathbf{x}_{\text{b}} - \mathbf{x}_j/m_+|^2}{\Delta^2_{\text{PSF},+}} \right] T(\mathbf{x}_{\text{b}})$$

or

$$\langle \hat{S}(\mathbf{x}_j) \rangle_{\text{quantum}} = K\Delta/(\pi w^2) \int d\mathbf{x}_{\text{b}} (\pi\sigma^2)^{-1} \exp\left[-\frac{|\mathbf{x}_{\text{b}} - \mathbf{x}_j|^2}{\sigma^2} \right] T(\mathbf{x}_{\text{b}})$$

or

$$\langle \hat{S}(\mathbf{x}_j) \rangle_{\text{quantum}} = K\Delta/(\pi w^2) \int d\mathbf{x}_{\text{b}}\, \delta(\mathbf{x}_{\text{b}} - \mathbf{x}_{\text{j}}) T(\mathbf{x}_{\text{b}})$$

or

$$\langle \hat{S}(\mathbf{x}_j) \rangle_{\text{quantum}} = K\,[\Delta/A]\; T(\mathbf{x}_{\text{j}})$$

Contrast

Quantum

Mean signal: $\langle \hat{S}(\mathbf{x}_j) \rangle_{\text{quantum}} = K \left[\Delta / A \right] \; T(\mathbf{x}_\text{j})$

No background

Contrast is infinite! (at least in principle)

Contrast

Quantum

Mean signal: $\langle\hat{S}(\mathbf{x}_j)\rangle_{\text{quantum}} = K\,[\Delta/A]\;T(\mathbf{x}_\mathrm{j})$

No background: Contrast is infinite

Thermal

$$\langle\hat{S}(\mathbf{x}_j)\rangle_{\text{thermal}} = \beta K \left(\frac{\overline{n}}{2}\right)^2 \Delta e^{-\mathbf{x}_j^2/\Delta^2_{\text{FOV},-}} \int d\mathbf{x}_\mathrm{b} \left(1 + \exp\left[-\frac{|\mathbf{x}_\mathrm{b} - \mathbf{x}_j/m_-|^2}{\Delta^2_{\text{PSF},-}}\right]\right) T(\mathbf{x}_\mathrm{b})$$

or

$$\langle\hat{S}(\mathbf{x}_j)\rangle_{\text{thermal}} = K \left(\frac{\overline{n}}{2}\right)^2 (\Delta/A^2) \int d\mathbf{x}_\mathrm{b} \left(1 + \exp\left[-\frac{|\mathbf{x}_\mathrm{b} - \mathbf{x}_j|^2}{\sigma^2}\right]\right) T(\mathbf{x}_\mathrm{b})$$

or

$$\langle\hat{S}(\mathbf{x}_j)\rangle_{\text{thermal}} = K \left(\frac{\overline{n}}{2}\right)^2 (\Delta/A^2)\,[A\overline{T} + \pi\sigma^2 T(\mathbf{x}_\mathrm{j})]$$

average transmission

(# of speckles transmitted through object)

Thus contrast is given by $$\frac{1}{[A/(\pi\sigma^2)]\overline{T}}$$

Roughly, number of pixels in ghost image

Noise

Consider image signal above bkgd:

$$\hat{S}'(\mathbf{x}) = \hat{S}(\mathbf{x}) - \hat{S}(\mathbf{x}_{\text{bkgd}})$$

$$T(\mathbf{x}_{\text{bkgd}}) = 0$$

Calculate:

$$\sigma^2_{\hat{S}'} = \left\langle \left[\hat{S}' - \langle \hat{S}' \rangle^2 \right]^2 \right\rangle$$

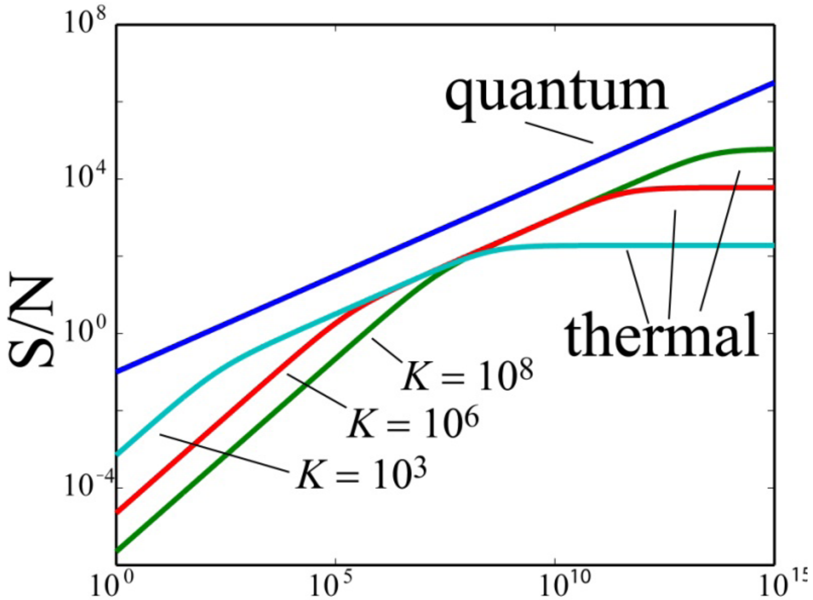
$$\sigma^2_{\hat{S}(\mathbf{x})} + \sigma^2_{\hat{S}_{\text{bkgd}}} - 2\sigma_{\hat{S},\hat{S}_{\text{bkgd}}}$$

$$\text{S/N} = \langle \hat{S}' \rangle / \sigma_{\hat{S}'}$$

Scaling Laws:

$$\text{S/N}_{\text{quantum}} \sim \sqrt{K}$$

$$\text{S/N}_{\text{thermal}} \sim \frac{\sqrt{K}\,\overline{n}\sigma^2}{[\alpha + \beta\overline{n} + \gamma\overline{n}^2]^{1/2}}$$



N is the total number of photons the illuminate object — $N = \overline{n}\,K$

(S/N) always worse for thermal (because of added noise from background

Conclusions

• For the same source parameters, quantum and thermal ghost images are resolved equally well.

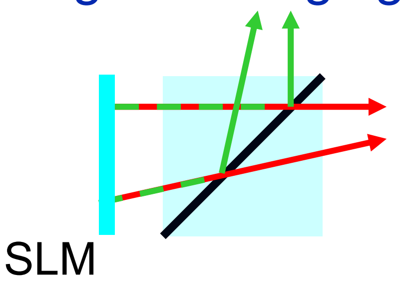
• Contrast of thermal ghost imaging degrades with increasing resolution.

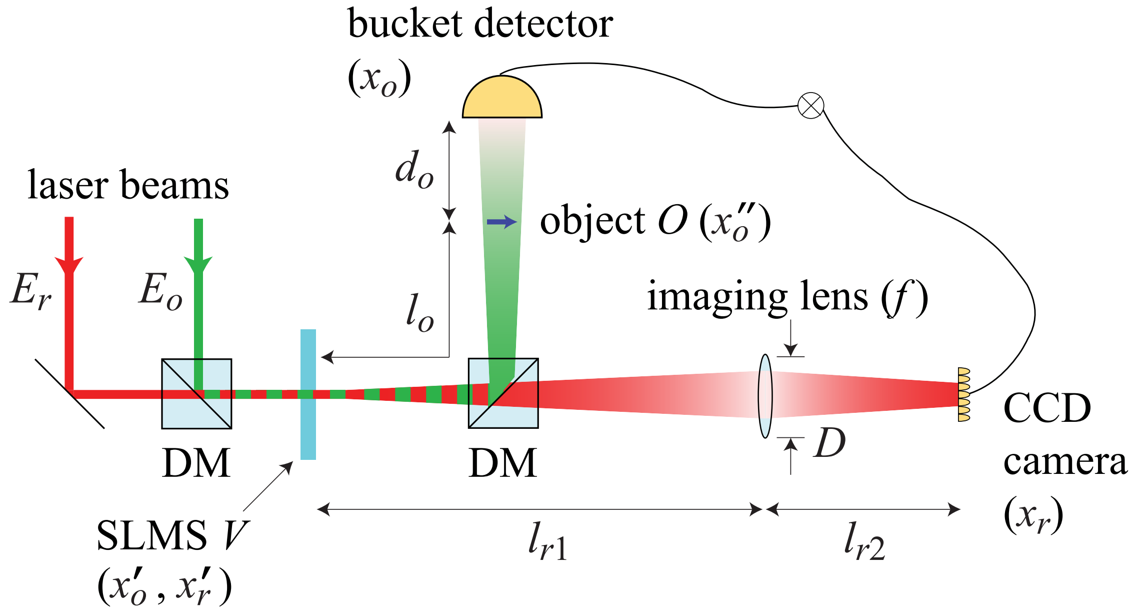
• S/N scales as $K^{1/2}$ (thermal ghost image degrades further at low photon numbers and high resolution).

Two-Color Ghost Imaging

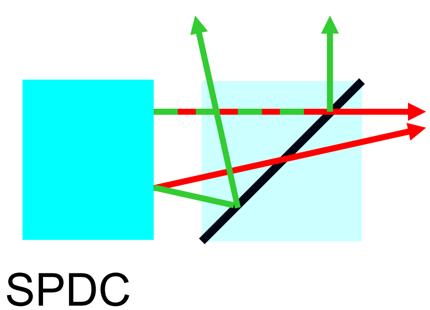
New possibilities afforded by using different colors in object and reference arms

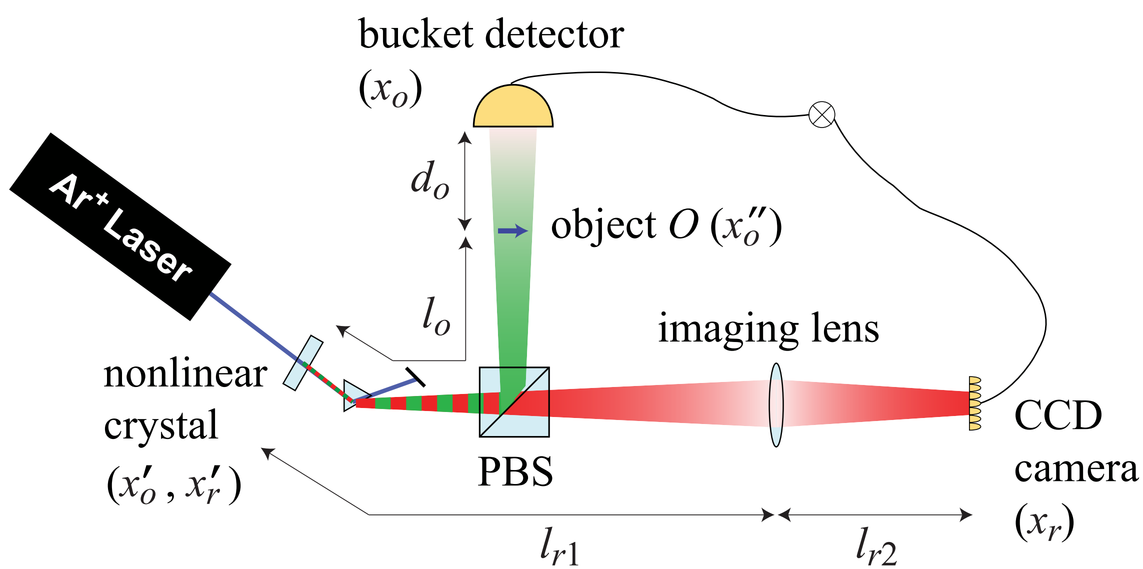
Thermal ghost imaging





Quantum ghost imaging





Chan, O'Sullivan, Boyd, PRA 2009

Two-Color Ghost Imaging: Model

- Classical (thermal): Gaussian-Schell Model

  Coherence function

$$W(\vec{x}'_o, \vec{x}'_r) = \exp\left[-\frac{\vec{x}'^{\,2}_o + \vec{x}'^{\,2}_r}{4w^2}\right] \exp\left[-\frac{\left(\vec{x}'_o - \vec{x}'_r\right)^2}{2\sigma_x^2}\right]$$

- Quantum (PDC): Gaussian approximation

  Two-photon wavefunction

$$\Psi(\vec{x}'_o, \vec{x}'_r) = \exp\left[-\frac{\vec{x}'^{\,2}_o + \vec{x}'^{\,2}_r}{4w^2}\right] \exp\left[-\frac{\left(\vec{x}'_o - \vec{x}'_r\right)^2}{2\sigma_x^2}\right]$$

w is the width of the laser beam; σ_x is the correlation distance (speckle size).

assume that $w \gg \sigma_x$

- Let D be the diameter of the imaging lens

Chan, O'Sullivan, Boyd, PRA 2009

Other Research Projects (Some Funded Separately)

Compressive sampling and ghost imaging
(with adaptive algorithms)

How fast to rotate the ground glass plate?

Exploiting the OAM degree of freedom

Propagation of quantum states through atmospheric turbulence

Materials for quantum lithography

Development of a single-photon source

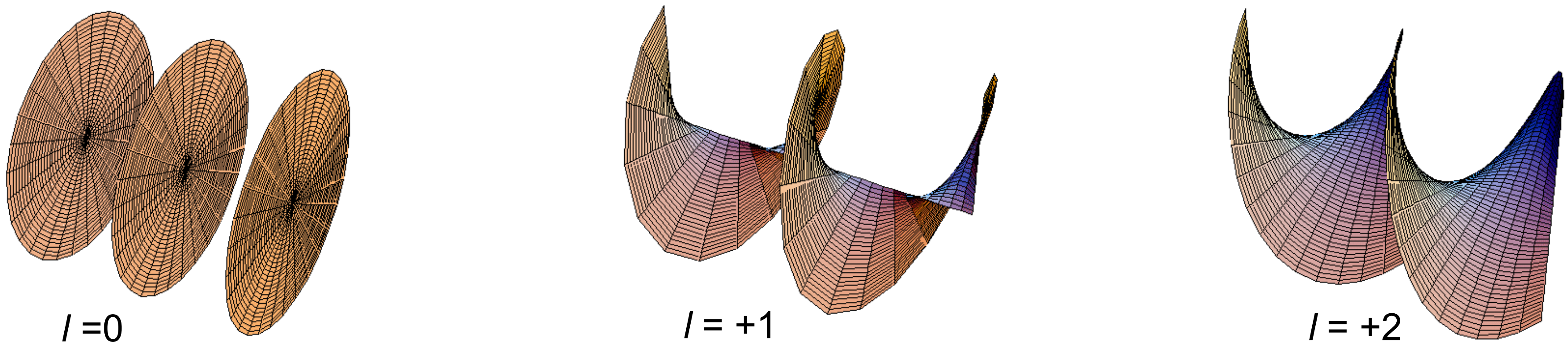
Use of the Orbital Angular Momentum of Light to Carry Quantum Information

Orbital angular momentum (OAM) spans an infinite-dimensional Hilbert space

Offers new potentialities for quantum information science

- How robust are the OAM states?
- Can we use them for free-space communications?
- How are they influenced by atmospheric turbulence?

Phase-front structure of some OAM states



J. Leach, J. Courtial, K. Skeldon, S. M. Barnett, S. Franke-Arnold and M. J. Padgett, Phys. Rev. Lett. 92, 013601 (2004).

A. Mair, A. Vaziri, G. Weihs and A. Zeilinger, Nature, 412, 313 (2001).

G. Molina-Terriza, J. P. Torres, and L. Torner, Phys. Rev. Lett. 88, 013601 (2002).

M. T. Gruneisen, W. A. Miller, R. C. Dymale and A. M. Sweiti, Appl. Opt. 47, A33 (2008).

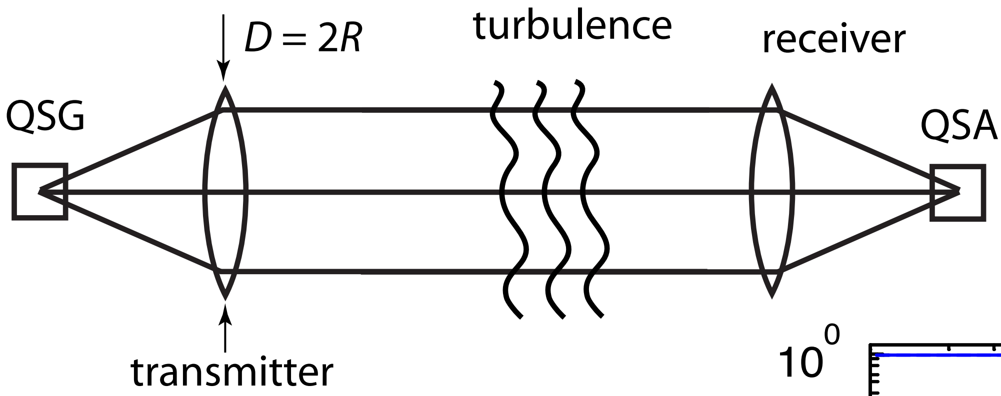
N. Gisin and R. Thew, Nature Photonics, 1, 165 (2007).

C. Paterson, Phys. Rev. Lett. 94, 153901 (2005).

C. Gopaul and R. Andrews, New J. of Physics, 9, 94 (2007).

G. Gbur and R. K. Tyson, J. Opt. Soc. Am. A, 25, 255 (2008).

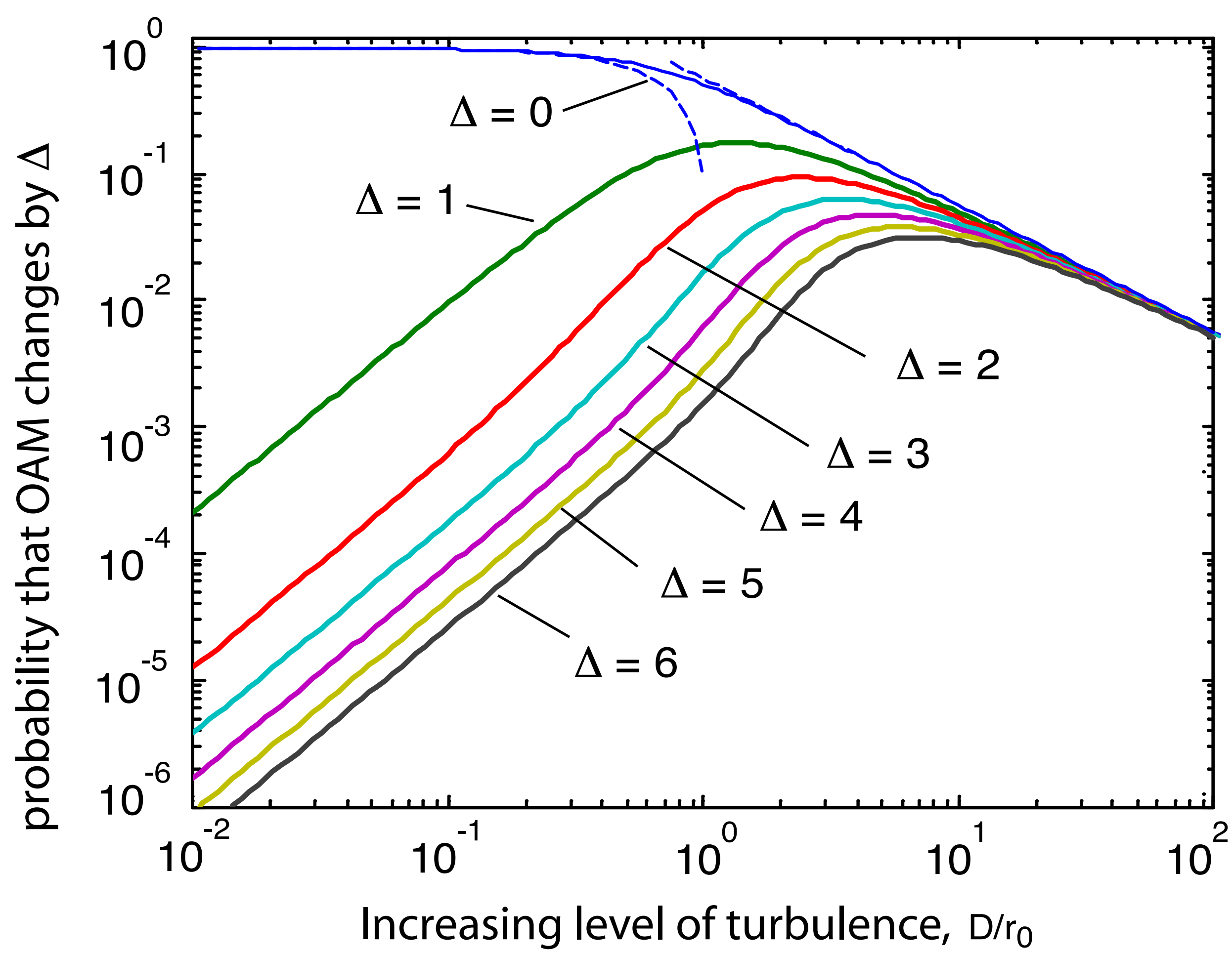
Influence of Atmospheric Turbulence on the Propagation of Quantum States of Light Carrying Orbital Angular Momentum



Probability that initial state is retained:

$$\langle s_0 \rangle = [1 + (1.845\, D/r_0)^2]^{-1/2}$$

r_0 = Fried parameter



Our results are qualitatively similar to those of Paterson (2005), but differ in detail because Paterson considered LG modes whereas we consider pure vortex beams. See also Smith and Raymer (2006).

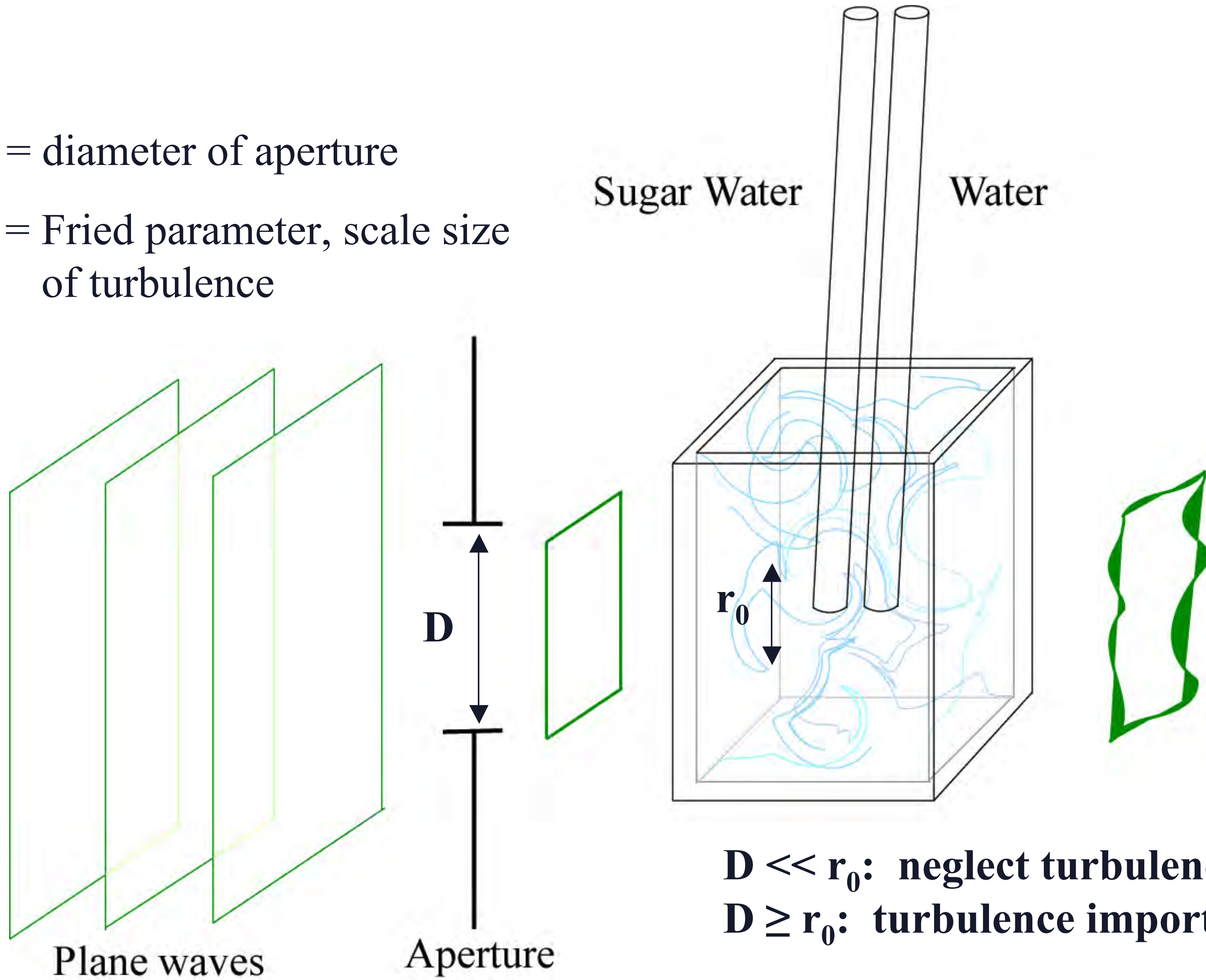
G. A. Tyler and R. W. Boyd, Opt Lett (2009).

Influence of Atmospheric Turbulence on the Quantum States of Light

To test these predictions in a laboratory setting, we have build a turbulence cell

D = diameter of aperture

$\mathbf{r_0}$ = Fried parameter, scale size of turbulence

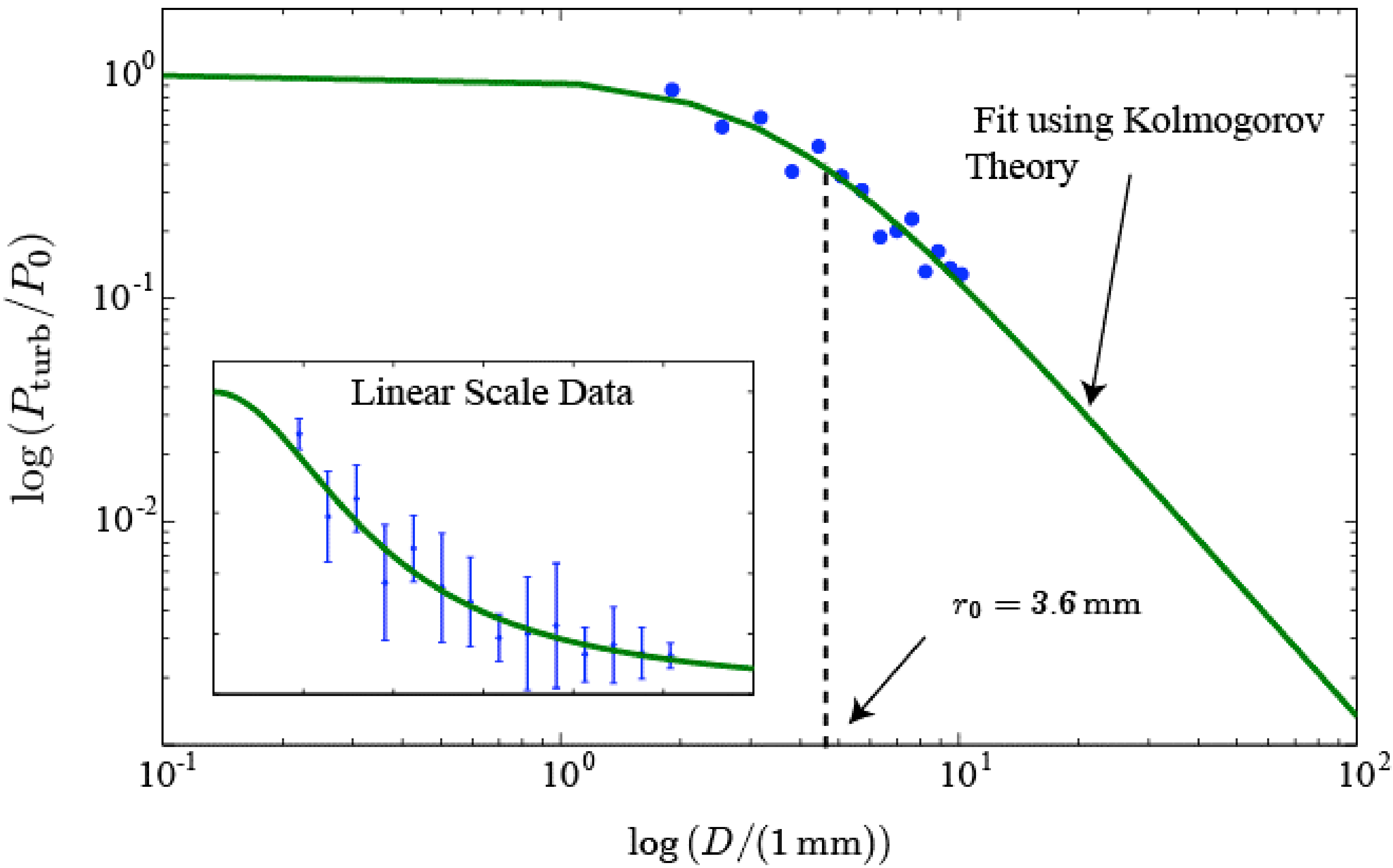


D << r_0: neglect turbulence

D ≥ r_0: turbulence important

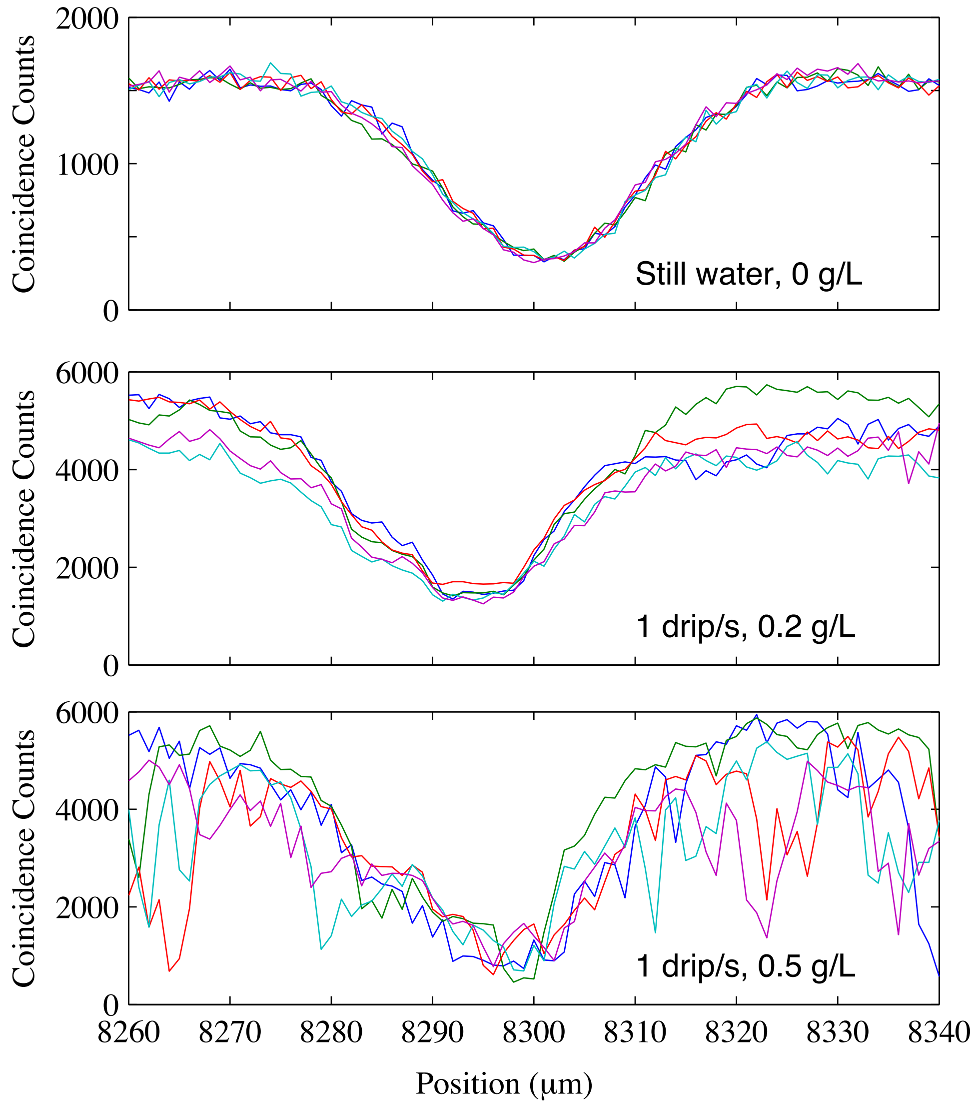
Influence of Atmospheric Turbulence on the Quantum States of Light

- Progress report: we are presently characterizing our turbulence cell
- As a first step, we measure the Strehl ratio as a function of beam diameter
- Strehl ratio is ratio of maximum beam intensity with and without turbulence
- Our data well modeled by Kolmogorov theory with r_0 = 3.6 mm



Influence of Atmospheric Turbulence on Quantum States of Light

- Recent result: How is the Hong-Ou-Mandel effect influenced by turbulence?
- Recall: The Hong-Ou-Mandel effect depends on the indistinguishability of the two interfering photons.
- Procedure: Place turbulence cell in one arm of Hong-Ou-Mandel interferometer



Tentative conclusion: turbulence leads to a loss of signal to noise ratio of the HOM effect, but does not influence the width or the depth of the Mandel dip.

Angular Two-Photon Interference and Angular Two-Qubit State

Anand Kumar Jha and Robert W. Boyd

The Institute of Optics, University of Rochester, Rochester, NY

Jonathan Leach, Barry Jack, Sonja Franke-Arnold, and Miles J. Padgett

Department of Physics and Astronomy, University of Glasgow, Glasgow, UK

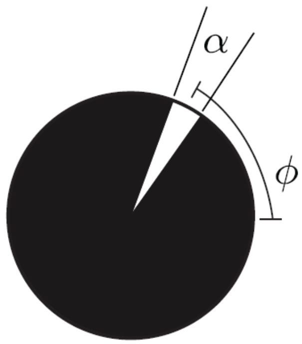
Stephen M. Barnett

Department of Physics, University of Strathclyde, Glasgow, UK

Symposium on Optical Interactions and Quantum Systems
University of Rochester, October 23 - 24, 2009

Angular Fourier Relationship

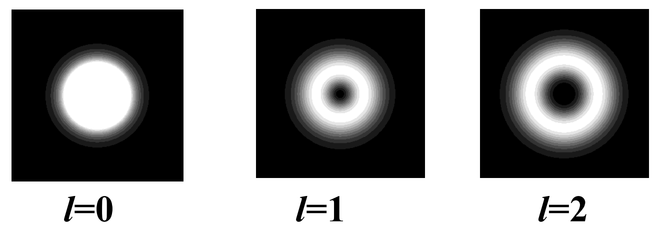
Angular position



Angular momentum

Laguerre-Gauss basis

LG_p^l with p=0



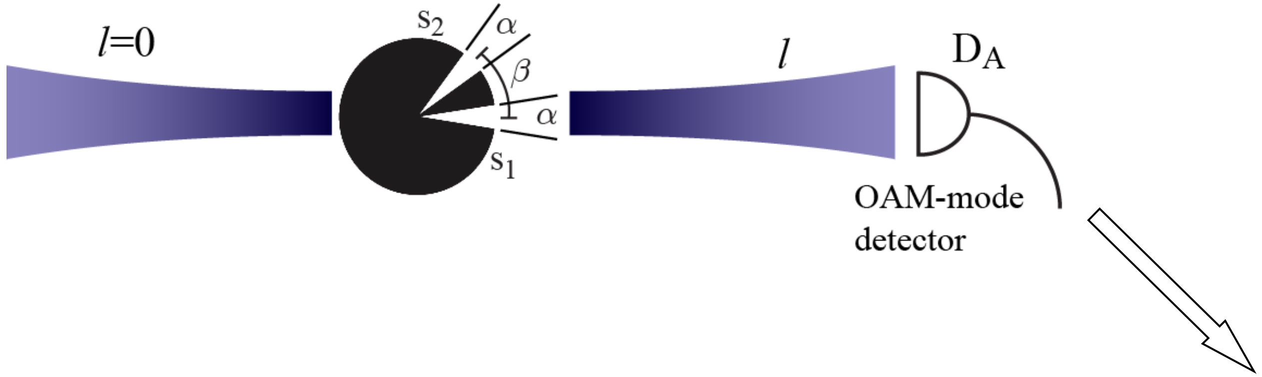
Allen et al., PRA **45**, 8185 (1992)

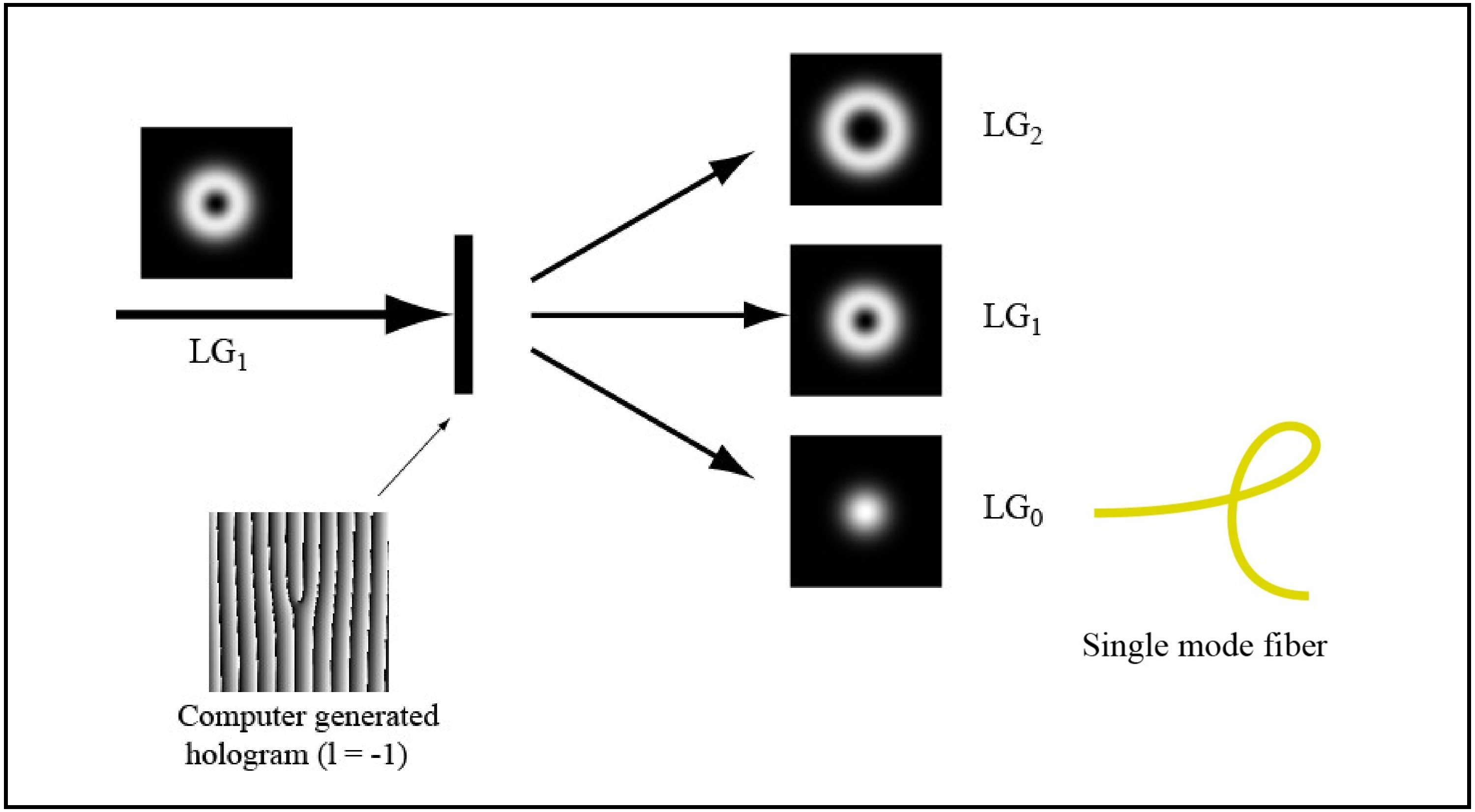
$$A_l = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi(\phi) \exp(-il\phi)$$

$$\Psi(\phi) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{+\infty} A_l \exp(il\phi)$$

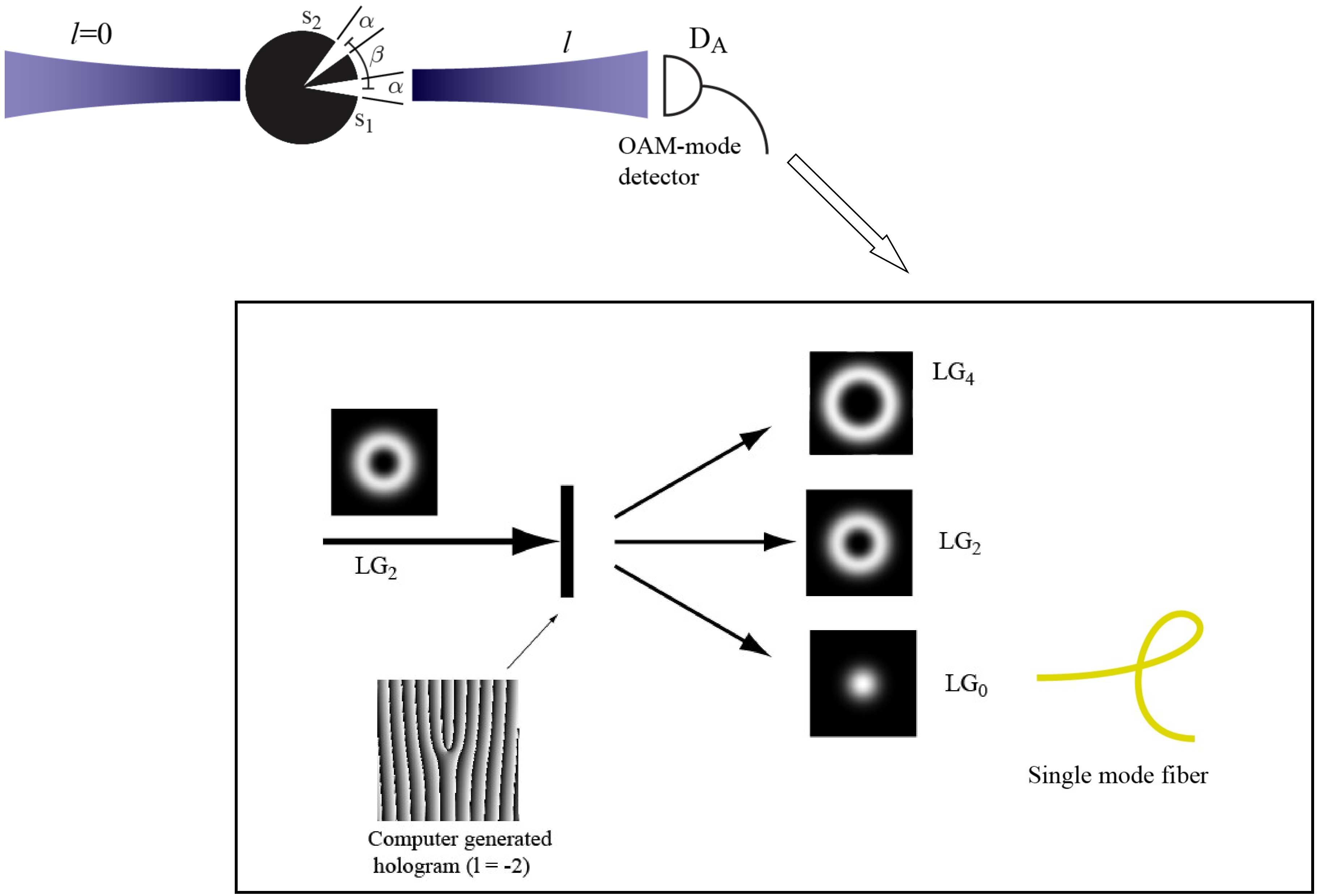
Barnett and Pegg, PRA **41**, 3427 (1990)
Franke-Arnold et al., New J. Phys. **6**, 103 (2004)
Forbes, Alonso, and Siegman J. Phys. A **36**, 707 (2003)

Angular One-Photon Interference

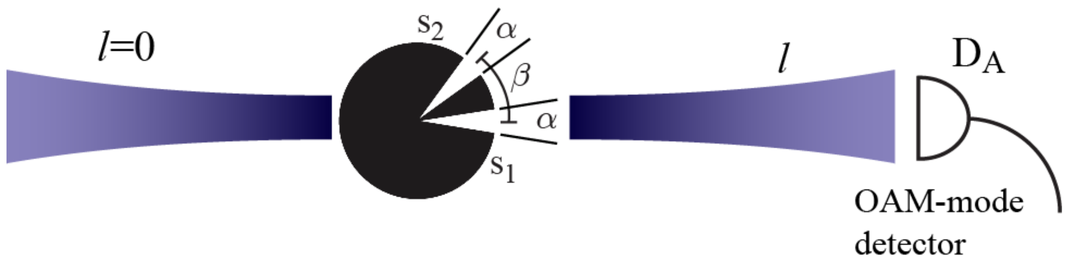


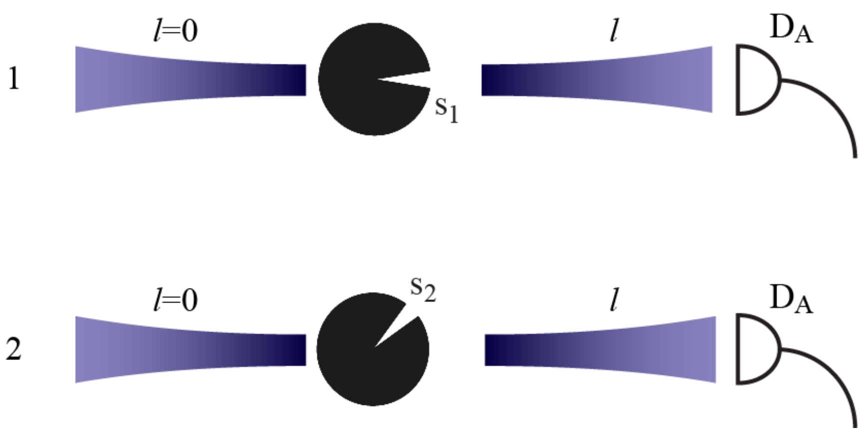


Angular One-Photon Interference



Angular One-Photon Interference



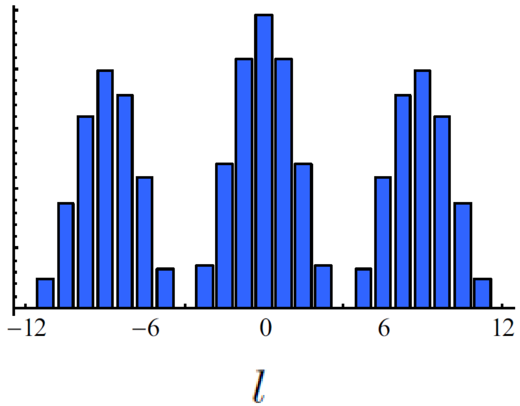


$$\psi_{1l} = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} d\phi \Psi_1(\phi) e^{-il\phi}$$

$$= \frac{\alpha}{\sqrt{2\pi}} \mathrm{sinc}\left(\frac{l\alpha}{2}\right)$$

$$\psi_{2l} = \frac{\alpha}{\sqrt{2\pi}} \mathrm{sinc}\left(\frac{l\alpha}{2}\right) e^{-il\beta}$$

$\alpha = \pi/10$
$\beta = \pi/4$

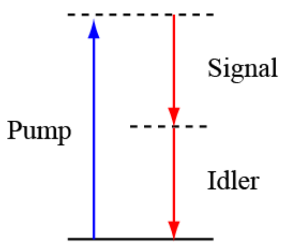


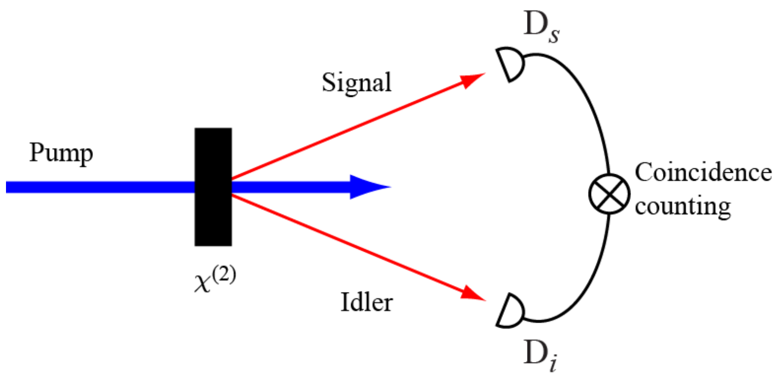
OAM-mode distribution:

$$I_A = C\frac{\alpha^2}{\pi} \mathrm{sinc}^2\left(\frac{l\alpha}{2}\right) [1 + \cos(l\beta)]$$

Yao et al., Opt. Express **14**, 13089 (2006)
Jha et al., PRA **78**, 043810 (2008)

Parametric down-conversion (PDC)





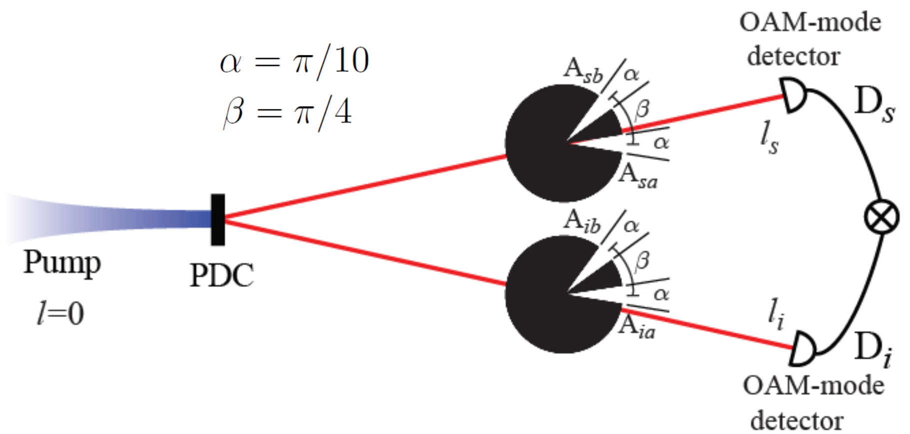
Burnham and Weinberg,
PRL **25**, 85 (1970)

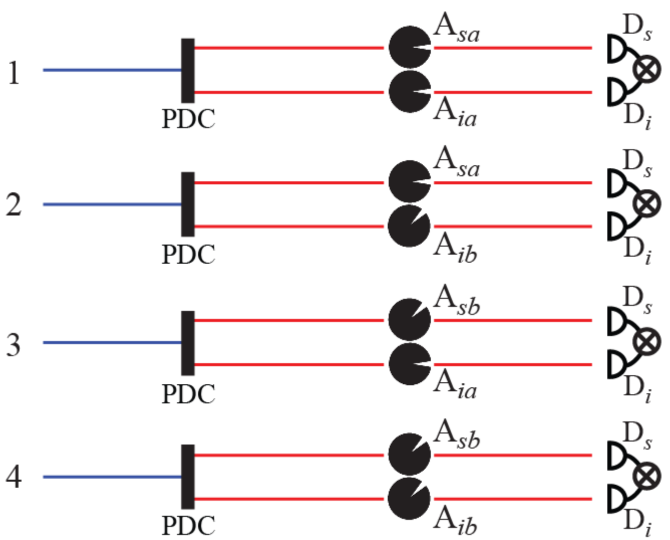
$$l_p = l_s + l_i$$

Entanglement in angular position and angular momentum

"Angular" two-photon interference

Angular Two-Photon Interference





State of the two photons produced by PDC:

$$|\psi_{\rm tp}\rangle = \sum_{l=-\infty}^{\infty} c_l |l\rangle_s |-l\rangle_i$$

State of the two photons after the aperture:

$$\rho_{\rm qubit} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}$$

$$\rho_{14} = \rho_{41}^* = \sqrt{\rho_{11}\rho_{44}}\ \mu e^{i\theta}$$

Coincidence count rate:

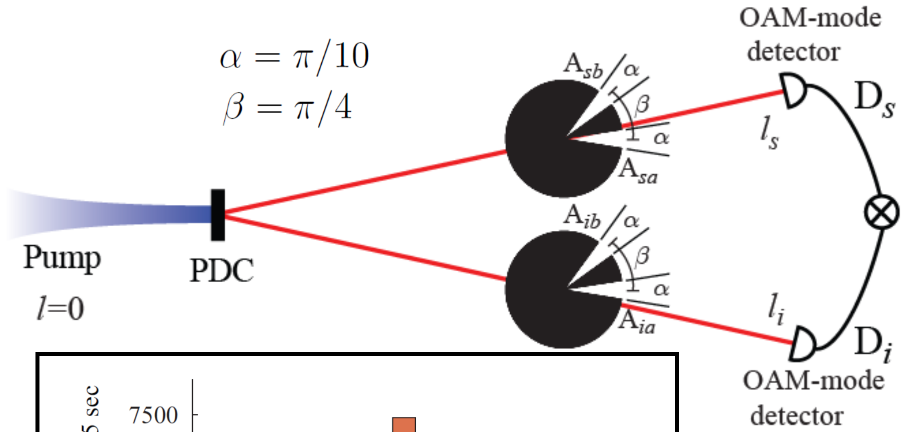
$$R_{si} = \frac{A^2\alpha^4}{4\pi^2}\left|\sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right]\right|^2 \times \{1 + 2\sqrt{\rho_{11}\rho_{44}}\ \mu \cos[(l_s + l_i)\beta + \theta]\}$$

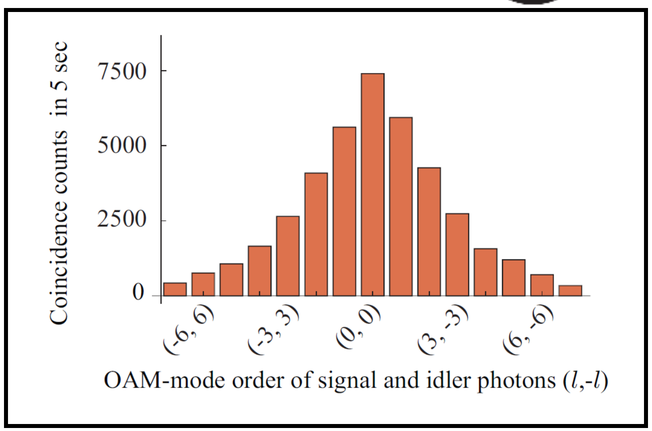
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

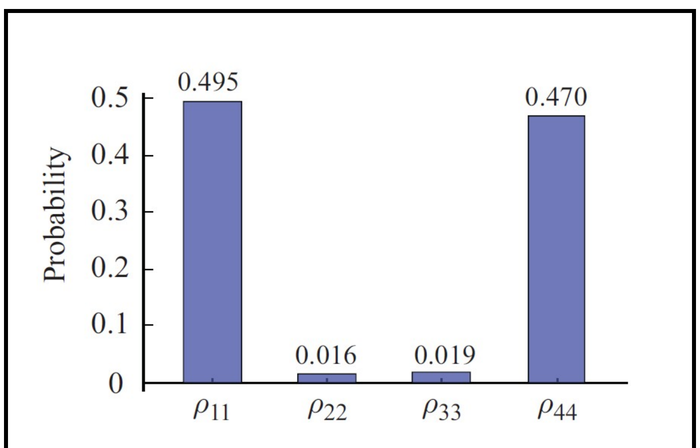
Concurrence of the two-qubit state:

$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\ \mu \ = V$$

Angular Two-Photon Interference







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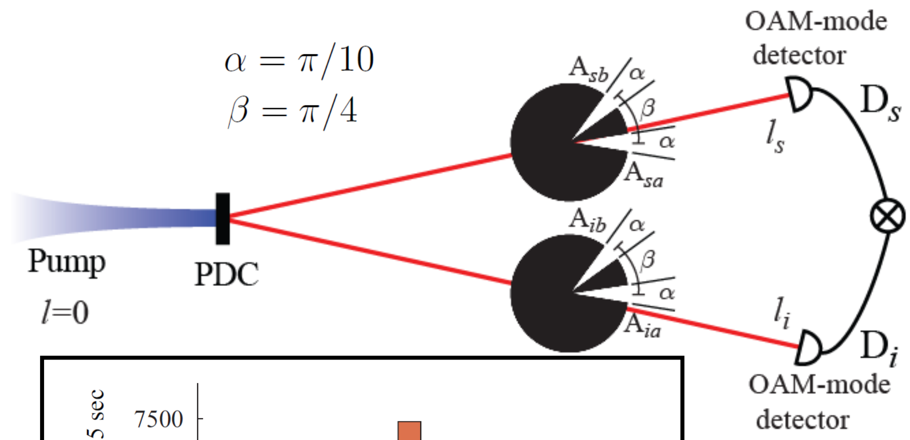
Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\ \mu$

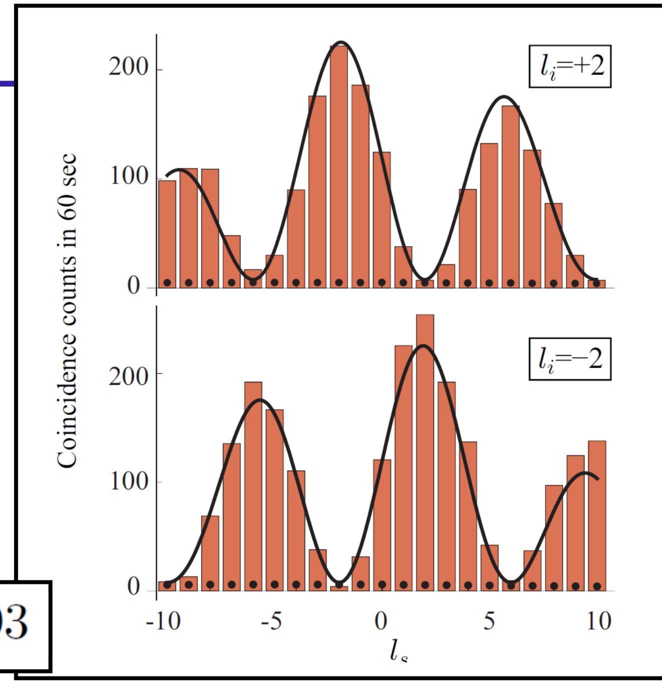
Concurrence of the two-qubit state:

$$C(\rho_{\rm qubit}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\ \mu = V$$

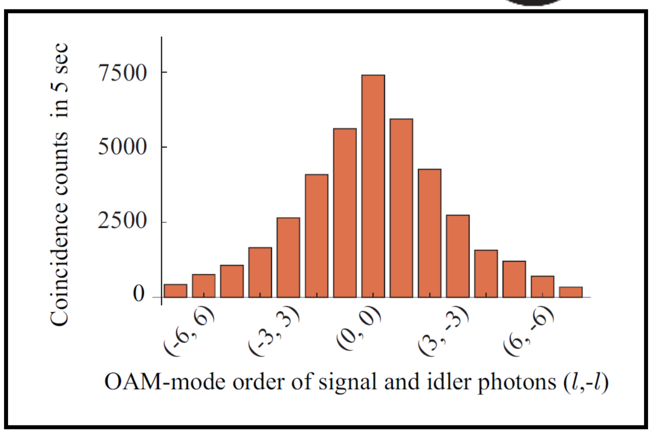
Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, submitted to PRL

Angular Two-Photon Interference





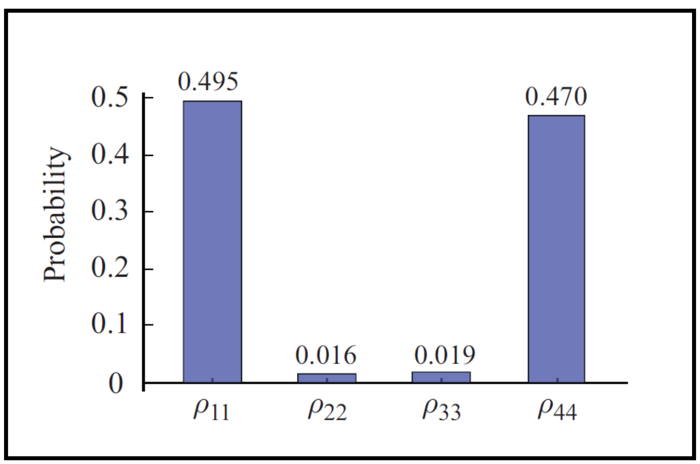
$C(\rho_{\text{qubit}}) = 0.93$



Coincidence count rate:

$$R_{si} = \frac{A^2\alpha^4}{4\pi^2} \left| \sum_l c_l \,\mathrm{sinc}\left[(l_s - l)\frac{\alpha}{2}\right] \mathrm{sinc}\left[(l_i + l)\frac{\alpha}{2}\right] \right|^2$$

$$\times \left\{1 + 2\sqrt{\rho_{11}\rho_{44}}\;\mu \cos\left[(l_s + l_i)\beta + \theta\right]\right\}$$



Visibility: $V = 2\sqrt{\rho_{11}\rho_{44}}\;\mu$

Concurrence of the two-qubit state:

$$C(\rho_{\text{qubit}}) = 2|\rho_{14}| = 2\sqrt{\rho_{11}\rho_{44}}\;\mu = V$$

Jha, Leach, Jack, Franke-Arnold, Barnett, Boyd, and Padgett, submitted to PRL

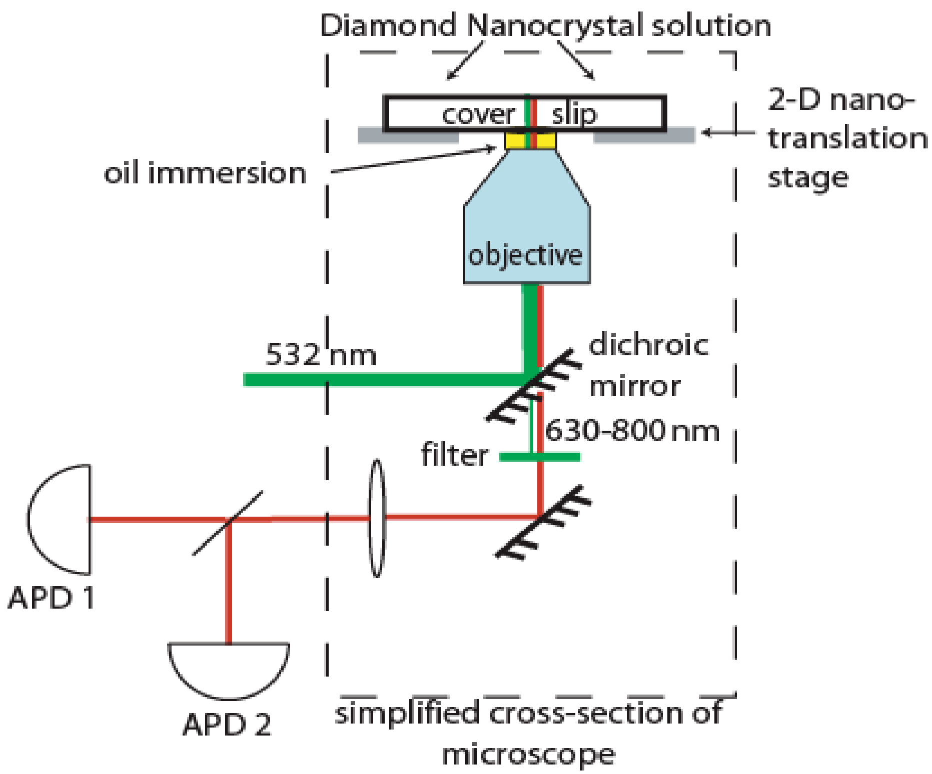
Conclusions

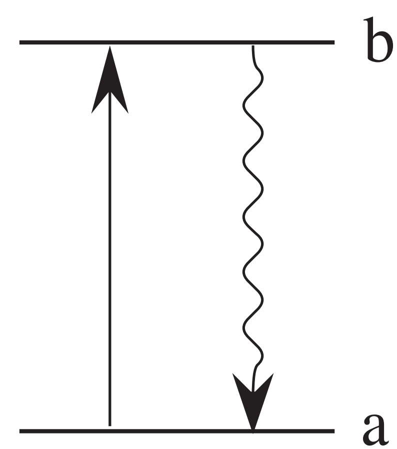
1. **Observation of angular two-photon interference effects**

2. **Demonstration of an angular two-qubit state**

An On-Demand Source of Single Photons

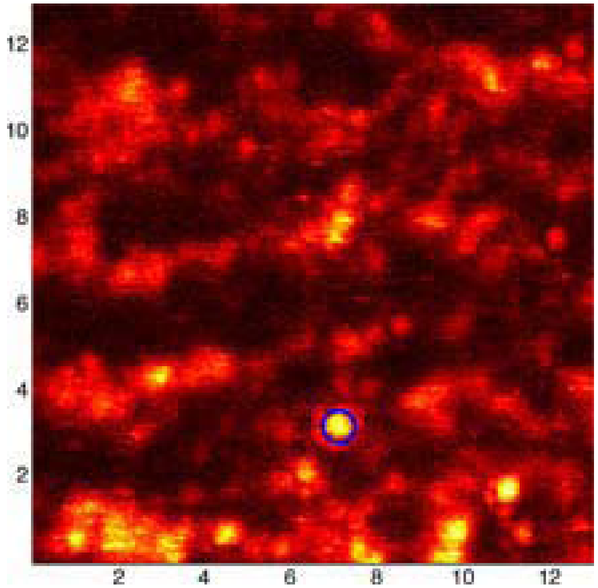
- Single-photon sources are crucial for many quantum-information protocols
- We make use of fluoresence from a single NV color center in diamond
- Our long-term goal is to embed the NV centers into chiral-nematic liquid crsytals
  - Fluorescence then can occur into only one polarization state



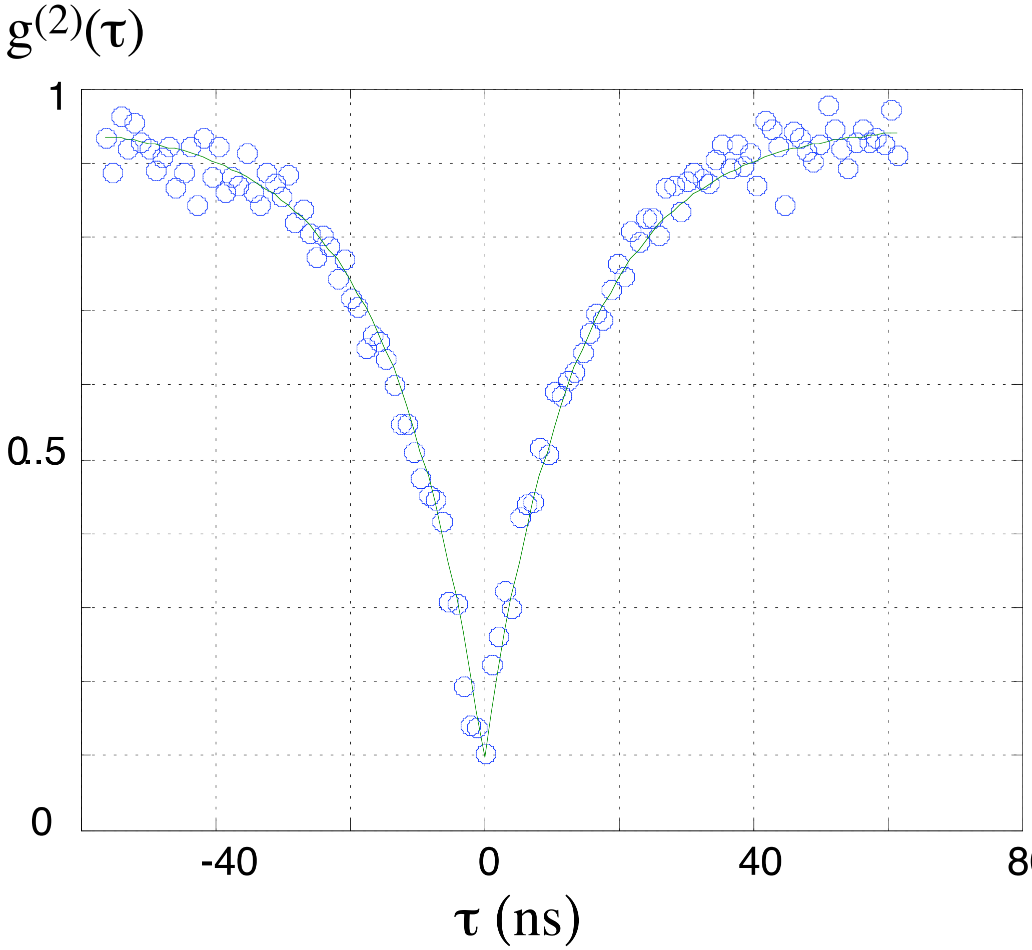


S. G. Lukishova, A. W. Schmid, R. Knox, P. Freivald, L. J. Bissell, R. W. Boyd, C. R. Stroud, Jr., and K. L. Marshall, J. Mod. Optics, 54 417 (2007)

Fluorescence antibunching of NV-color centers in nanodiamonds



diamond nanocrystals approximately 25 nm in diameter

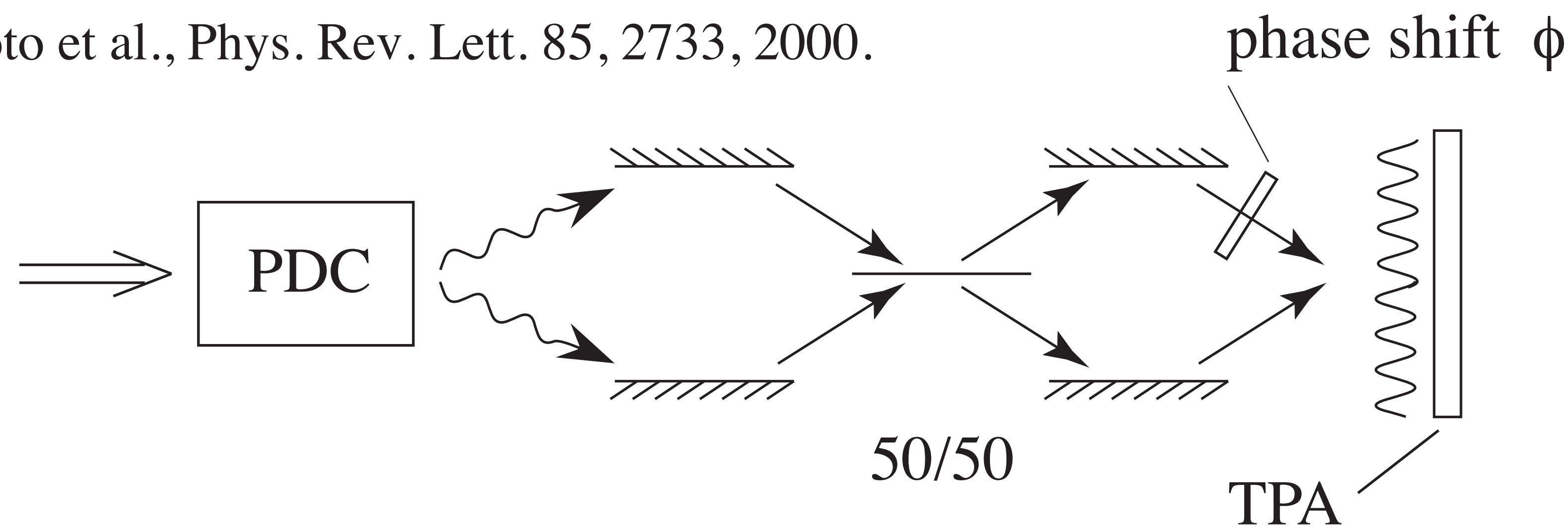


Data show that photons are emitted one at a time!

Quantum Lithography

- Entangled photons can be used to form an interference pattern with detail finer than the Rayleigh limit
- Process "in reverse" performs sub-Rayleigh microscopy, etc.
- Resolution ≈ $\lambda / 2N$, where N = number of entangled photons

Boto et al., Phys. Rev. Lett. 85, 2733, 2000.



- No *compelling* laboratory demonstration to date
- Primary difficulty: need extremely sensitive recording material

Quantum Lithography – Materials Issues

What are the sensitivities of typical recording materials?

Silver halide holographic plates: 1 mJ/cm^2

Dichromated gelatin holographic plates: 100 mJ/cm^2

Two-photon photopolymer (Kawata): 1 MJ/cm^2

What typical values of multiphoton cross sections?

$\sigma^{(2)}$ typically 1 GM where 1 GM = 10^{-50} cm^2 s/photon

For a very good two-photon absorber, $\sigma^{(2)} = 1000$ GM

We estimate that for PMMA $\sigma^{(3)} = 10^{-85}$ cm^4 s^2/photon

Can we do even better?

Good evidence that $\sigma^{(2)}$ and $\sigma^{(3)}$ can be enhanced by as much as 500-fold by coupling to a plasmonic resonance!

[Kano and Kawata, Opt. Lett, 21, 1848 1996;
Cohanoschi and Hernández, J. Phys. Chem. B 109, 14506 2005]

Observation of a Microscopic Cascaded Contribution to the Fifth-Order Nonlinear Susceptibility

Ksenia Dolgaleva, Heedeuk Shin, and Robert W. Boyd,

The Institute of Optics
University of Rochester

http://www.optics.rochester.edu/~boyd

Phys. Rev. Lett. 103, 113902 (2009).

Cascading in Nonlinear Optics

Example:

$$\chi_{\text{eff}}^{(3)} = \text{const} \times \chi^{(2)} \cdot \chi^{(2)}$$

Two types of cascading:

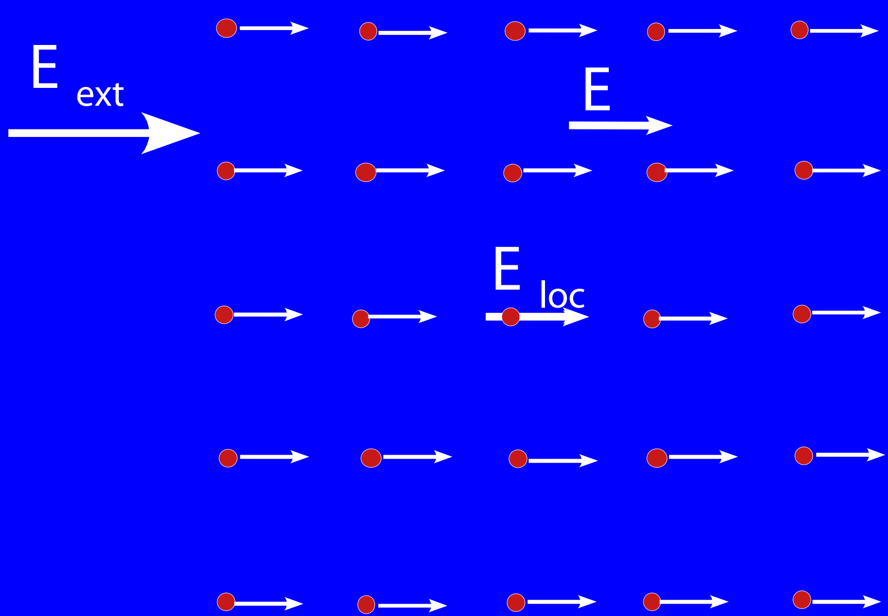
- Microscopic cascading:
  results from local-field effects (dipole-dipole coupling)
- Macroscopic cascading:
  results from propagation effects

Motivation

- Why high-order nonlinearities?
  - High-order NLO
  - Quantum Information Science
    (to detect *N*-photon states)
  - And specifically, for quantum lithography
    (Boto et al. 2002)
- Why microscopic cascading?
  - Lower-order nonlinear effects are stronger
  - Cascading should be of local nature

Local Field Effects in Nonlinear Optics

Consider a medium exposed to
an external optical field:



$$E_{loc} \neq E_{ext} \neq E$$

Local field is responsible
for optical properties!

E - macroscopic average field

E_{loc} - local field

Lorentz Local Field

$$E_{loc} = E + \frac{4\pi}{3} P \qquad \text{or} \qquad E_{loc} = L\, E$$

where

$$L = \frac{\epsilon^{(1)} + 2}{3}$$

is Lorentz local-field correction factor

$\epsilon^{(1)}$ - dielectric permittivity

Local Field in Nonlinear Optics

Local-field-corrected

$$P = \chi E = \chi^{(1)} E + 3\chi^{(3)} |E|^2 E + 10\chi^{(5)} |E|^4 E + \ldots$$

(Assumes a centrosymmetric medium)

Common Misconception

Since

$$\chi^{(3)} = N\,\gamma_{at}^{(3)} |L|^2 L^2,$$

one would think that

$$\chi^{(5)} = N\,\gamma_{at}^{(5)} |L|^4 L^2.$$

But this is not correct!

Microscopic Cascading by Local-Field Effects

direct contribution from fifth-order hyperpolarizability $\gamma_{at}^{(5)}$

$$\chi^{(5)} = N\,\gamma_{at}^{(5)}\,|L|^{4}L^{2}$$

$$\frac{24\pi}{10} N^{2}\left(\gamma_{at}^{(3)}\right)^{2}|L|^{4}L^{3} \quad \frac{12\pi}{10} N^{2}\left|\gamma_{at}^{(3)}\right|^{2}|L|^{6}L\,.$$

microscopic cascaded contributions from third-order hyperpolarizability $\gamma_{at}^{(3)}$

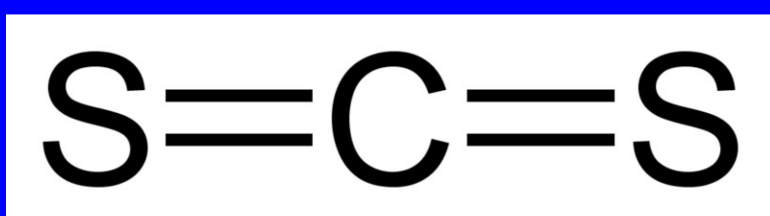
Note N^2 scaling!

Dolgaleva, Boyd, Sipe, PRA (2007)

Experimental Identification of the Microscopic Cascaded Contribution to $\chi^{(5)}$

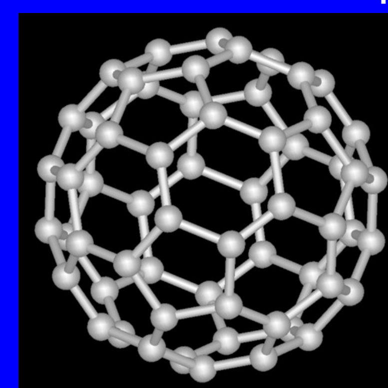
Changing the concentration of fullerene C_{60} in CS_2, we measured $\chi^{(5)}$ as a function of N .

Carbon disulfide

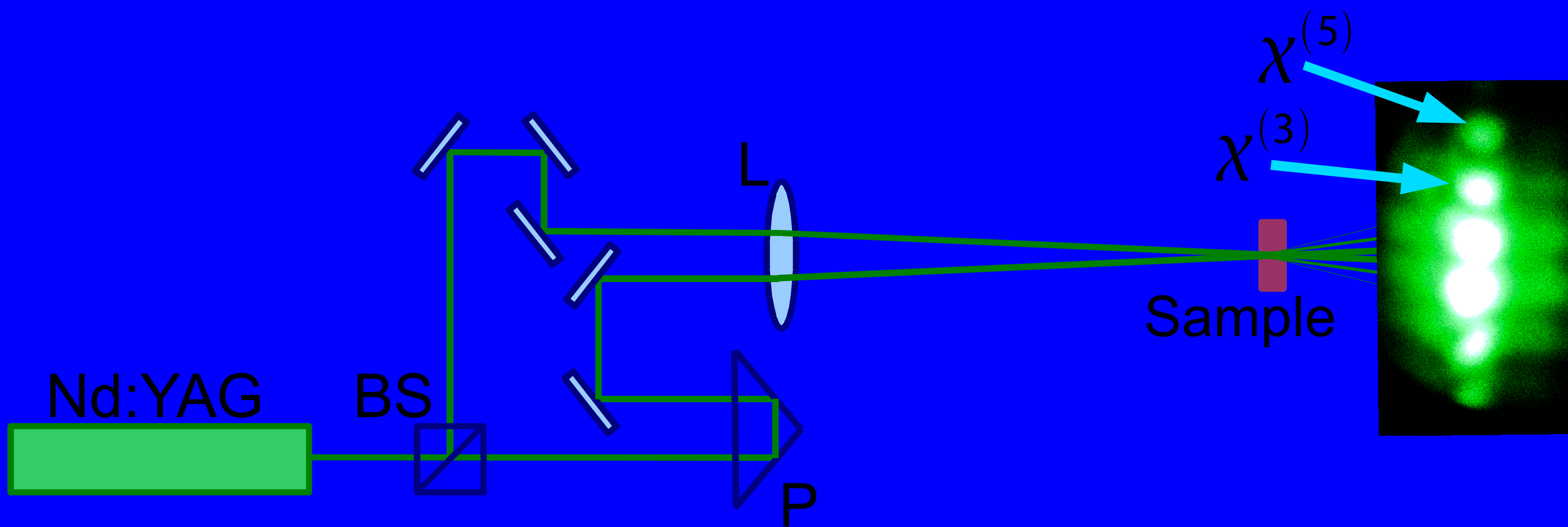


+

Fullerene $C_{:6}$



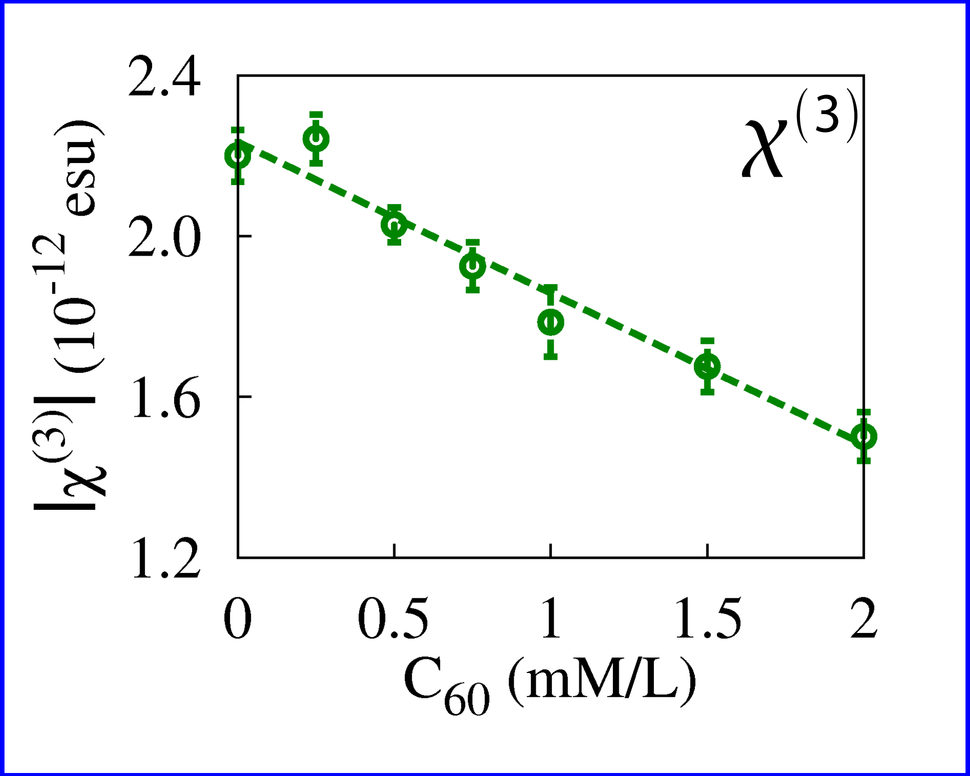
Experimental Setup

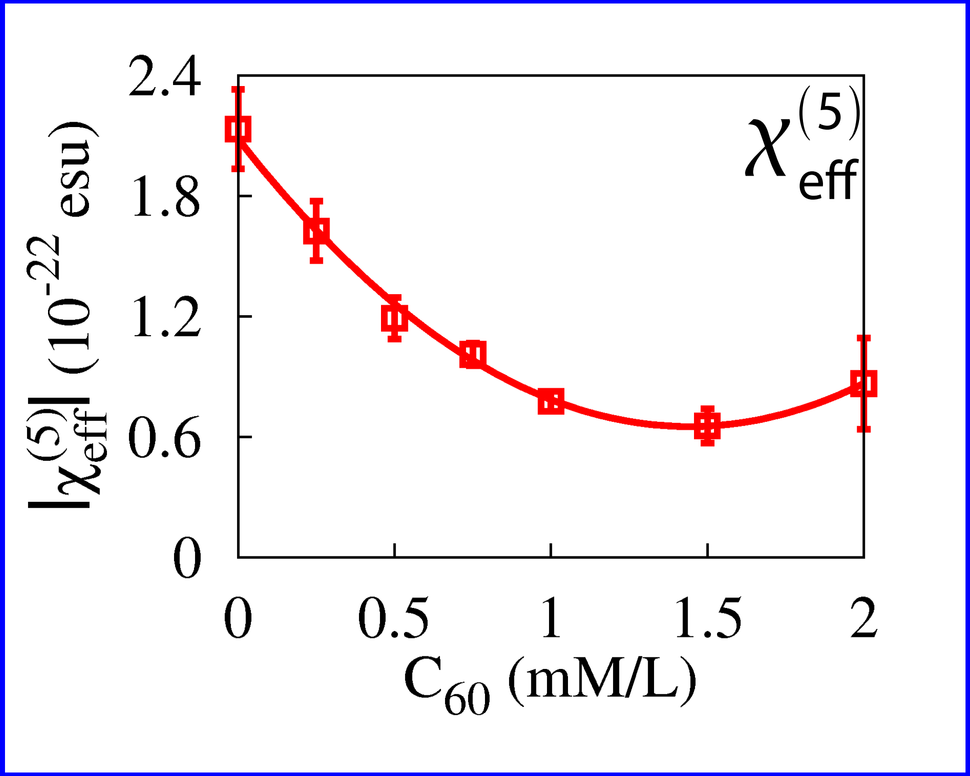


35 ps, 10 pps, 532 nm

Nonlinear Susceptibilities as Functions of C_{60} Molar Concentration

C_{60} and CS_2 have nonlinear responses of opposite signs.



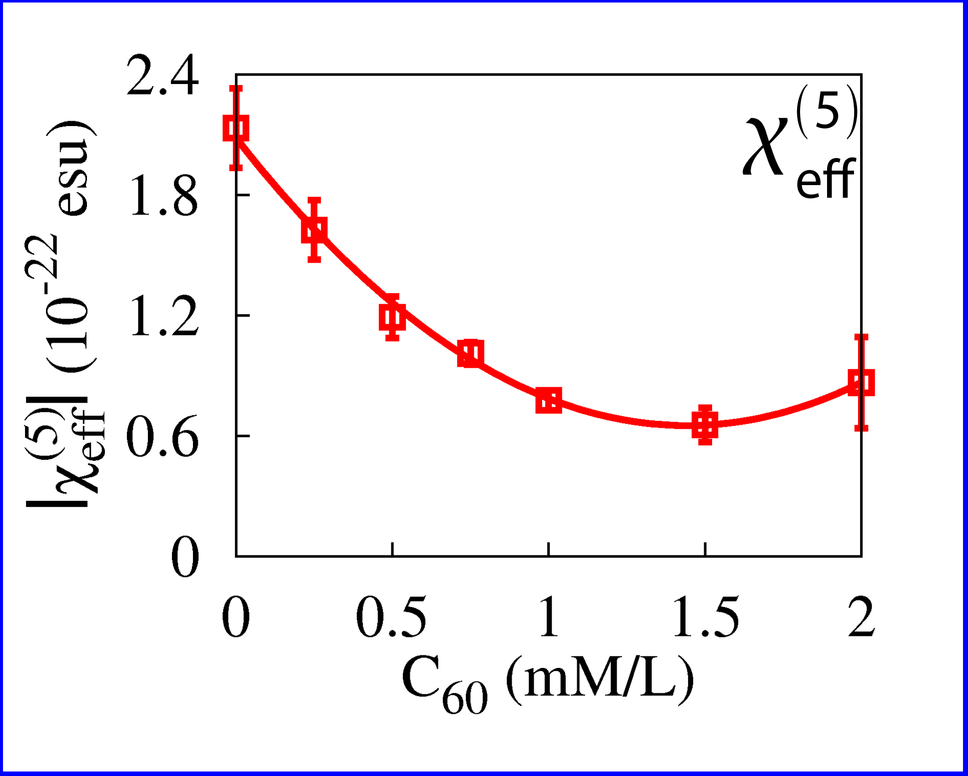


$\chi^{(5)}$ as a Functions of C_{60} Molar Concentration

$$\chi^{(5)}(esu) = 2.1\times10^{-22} - 2.0\times10^{-22}\,N \quad 6.9\times10^{-23}\,N^{2}$$

$$\chi^{(5)} = \chi^{(5)}_{dir} + \chi^{(5)}_{micro}$$

$\chi^{(5)}_{dir} \rightarrow N$ $\qquad$ $\chi^{(5)}_{micro} \rightarrow N^{2}$



But: $\chi^{(5)}_{total} = \chi^{(5)}_{dir} + \chi^{(5)}_{casc}$

$$\chi^{(5)}_{casc} \rightarrow \chi^{(5)}_{micro} + \chi^{(5)}_{macro} \propto N^{2}$$

Separating Microscopic and Macroscopic Cascaded Contributions

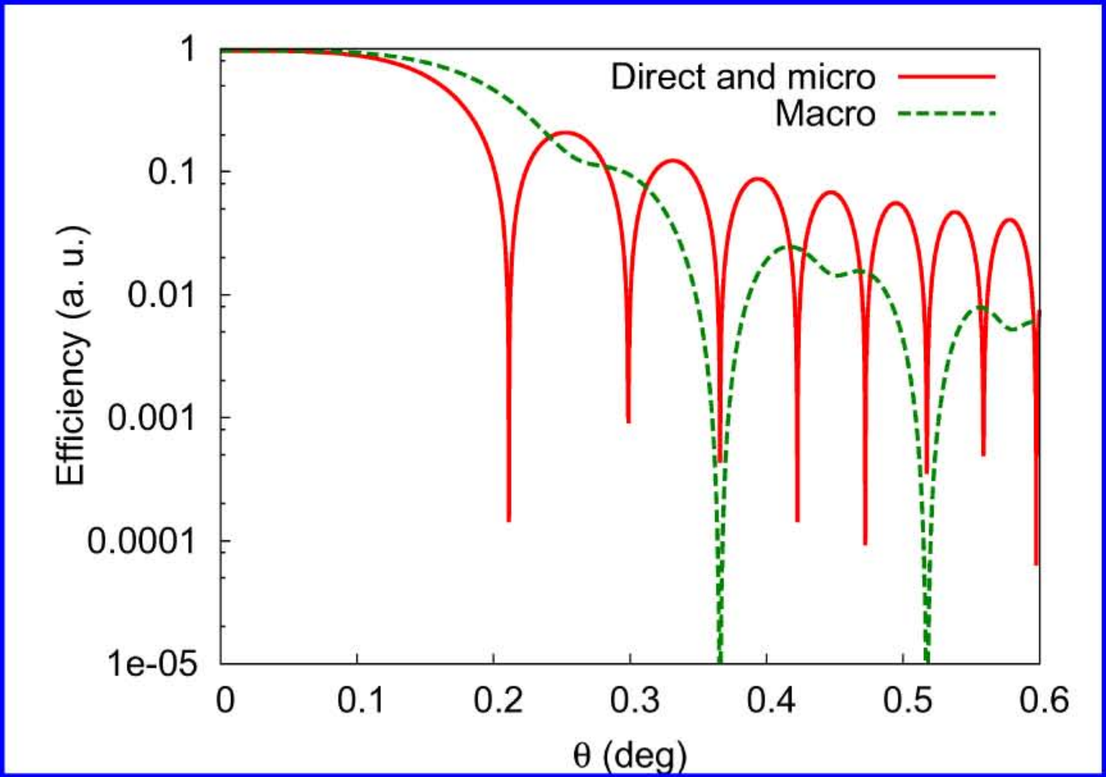
Solving the driven wave equation

$$\nabla^2 \tilde{E} - \frac{\epsilon}{c^2}\frac{\partial^2 \tilde{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 \tilde{P}}{\partial t^2},$$

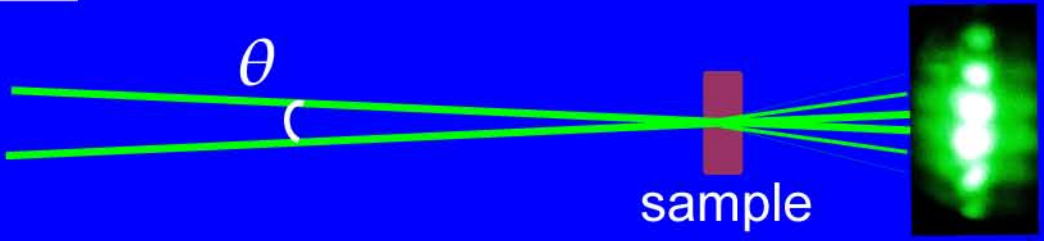
where $\tilde{E} = A(r)e^{i(kr-\omega t)} \quad c.\ c.,$

for microscopic (direct and cascaded) and macroscopic cascaded contributions separately, we found that these contributions have different phase mismatch'

Efficiency of the Microscopic and Macroscopic Contributions

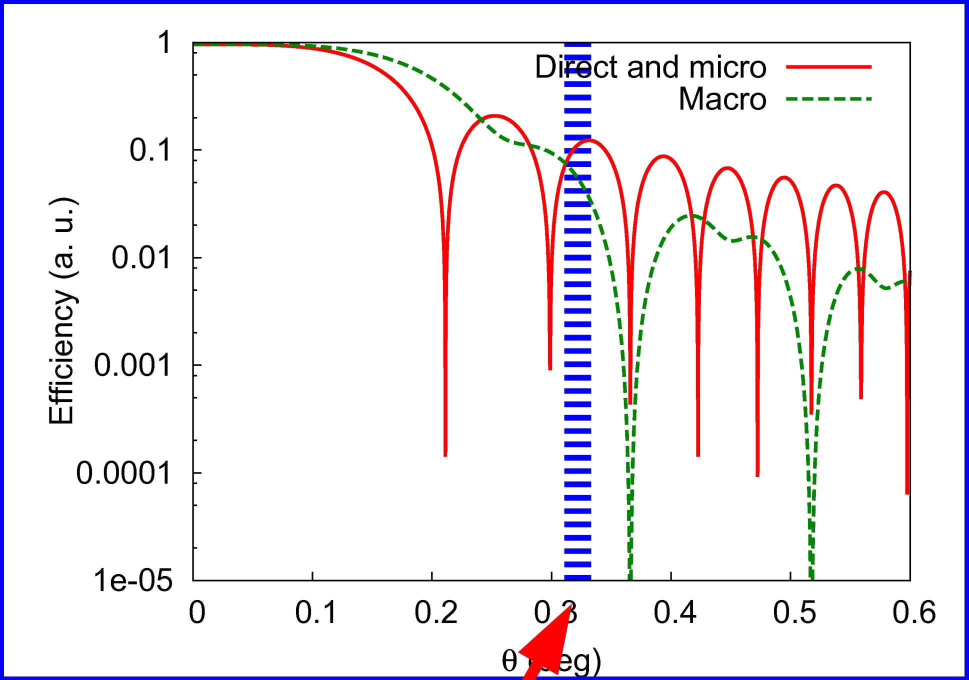


$$\chi^{(5)}_{macro} \propto \left(\chi^{(3)}\right)^2$$



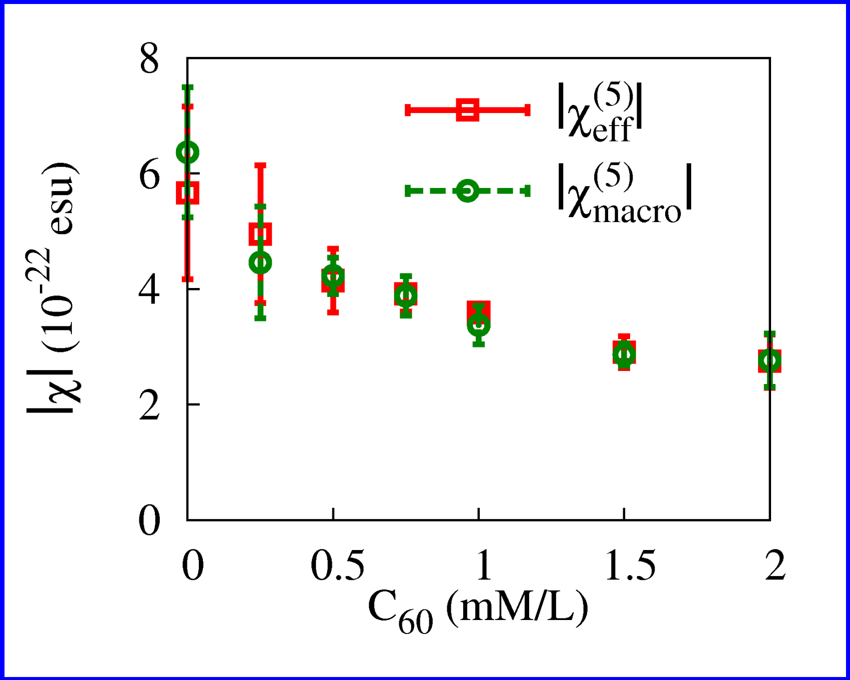
Separating the Microscopic and Macroscopic Contributions

Efficiency of the contributions



Measurement was done for this angle

$\chi^{(5)}_{eff}$ and $\chi^{(5)}_{macro}$



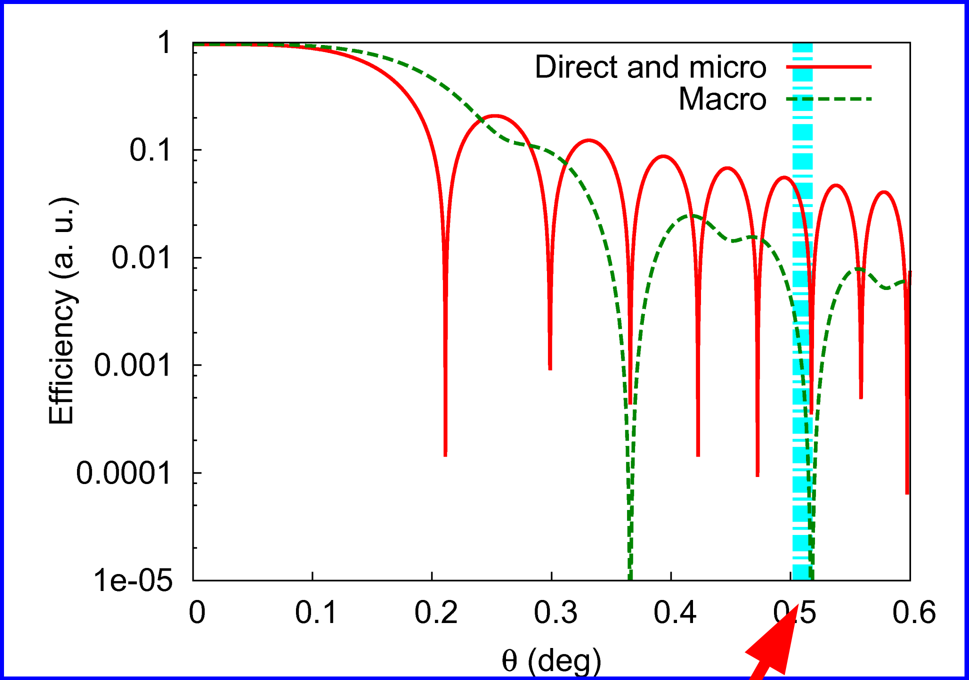
Note: $\chi^{(5)}_{macro} \propto [\chi^{(3)}]^2$

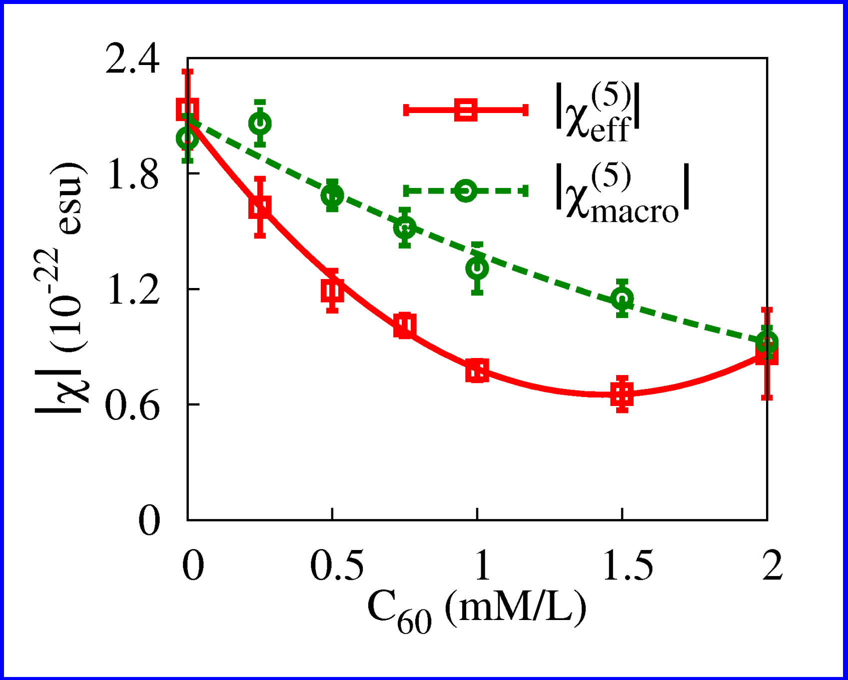
For this angle,

$\chi^{(5)}$ is due entirely to $\chi^{(5)}_{macro}$

Separating the Microscopic and Macroscopic Contributions

Efficiency of the contributions $\chi^{(5)}_{eff}$ and $\chi^{(5)}_{macro}$





Measurement was done for this angle

Microscopic cascaded contribution is about a factor of 10 larger than the macroscopic cascaded contribution.

ROBERT BOYD -- ACCOMPLISHMENTS 2008-2008

Published Papers

B1. Temporal coherence and indistinguishability in two-photon interference effects, A. K. Jha, M. N. O'Sullivan, K. W. C. Chan, and R. W. Boyd, Phys. Rev. A 77, 021801 (2008).

B2. Conditional preparation of states containing a definite number of photons, M. N. O'Sullivan, K. W. C. Chan, V. Lakshminarayanan, and R. W. Boyd, Phys. Rev. A 77, 023804 (2008).

B3. Propagation of Quantum States of Light through Absorbing and Amplifying Media, R.W. Boyd, G.S. Agarwal, K.W.C. Chan, A.K. Jha, and M.N. O'Sullivan, in Optics Communications, 281, 3732 (2008).

B4. Let Quantum Mechanics Improve Your Images, R.W. Boyd, Science, 321, 501 (2008).

B5. Exploring Energy-Time Entanglement using Geometric Phase, A.K. Jha, M. Malik, and R.W. Boyd, Phys. Rev. Lett. 101, 180405 (2008).

B6. Two-Color Ghost Imaging, K.W.C. Chan, M.N. O'Sullivan, and R.W. Boyd, Phys. Rev. A 79, 033808 (2009).

B7. Discriminating Orthogonal Single-Photon Images, C. J. Broadbent, P. Zerom, H. Shin, J. C. Howell, and R. W. Boyd Phys. Rev. A 79 033802 (2009).)

B8. Fourier Relation between the Angle and Angular Momentum for Entangled Photons, A. K. Jha, B. Jack, E. Yao, J. Leach, R.W. Boyd, G.S. Buller, S.M. Barnett, S. Franke-Arnold, and M.J. Padgett, Phys. Rev. A 78, 043810 (2008).

B9. Violation of a Bell inequality in two-dimensional orbital angular momentum state spaces, J. Leach, B. Jack, J. Romero, M. Ritsch-Marte, R. W. Boyd, A. K. Jha, S. M. Barnett, S. Franke-Arnold, and M. J. Padgett, Opt. Express, 17, 8287 (2009).

B10. Observation of a Microscopic Cascaded Contribution to the Fifth-Order Nonlinear Susceptibility, K. Dolgaleva, H. Shin, and R. W. Boyd, Phys. Rev. Lett. 103, 113902 (2009).

B11. Angular Two-Qubit States and Two-Photon Angular Interference, A. K. Jha, J. Leach, B. Jack, S. Franke-Arnold, S. M. Barnett, R W. Boyd, and M J. Padgett, in review.

B12. High-Order Thermal Ghost Imaging, K. W. C. Chan, M. N. O'Sullivan, and R. W. Boyd, in review.

Departmental Colloquia, etc.

Los Alamos National Laboratory
University of New Mexico
University of Erlangen
Danish Technical University

Talks at international Conferences

PQE, Snowbird Utah
SPIE DSS, Orlando
OASIS (an Israeli conference somewhat similar to CLEO), Tel Aviv
Photonics West, San Jose
SPIE Annual Meeting, San Diego
ICSSUR, Olomouc, Czech Republic
OSA Annual Meeting, Rochester
IQEC, Baltimore
Nonlinear Optics (an OSA conference)

Graduated PhD Students

Ksenia Dolgaleva
Giovanni Piredda

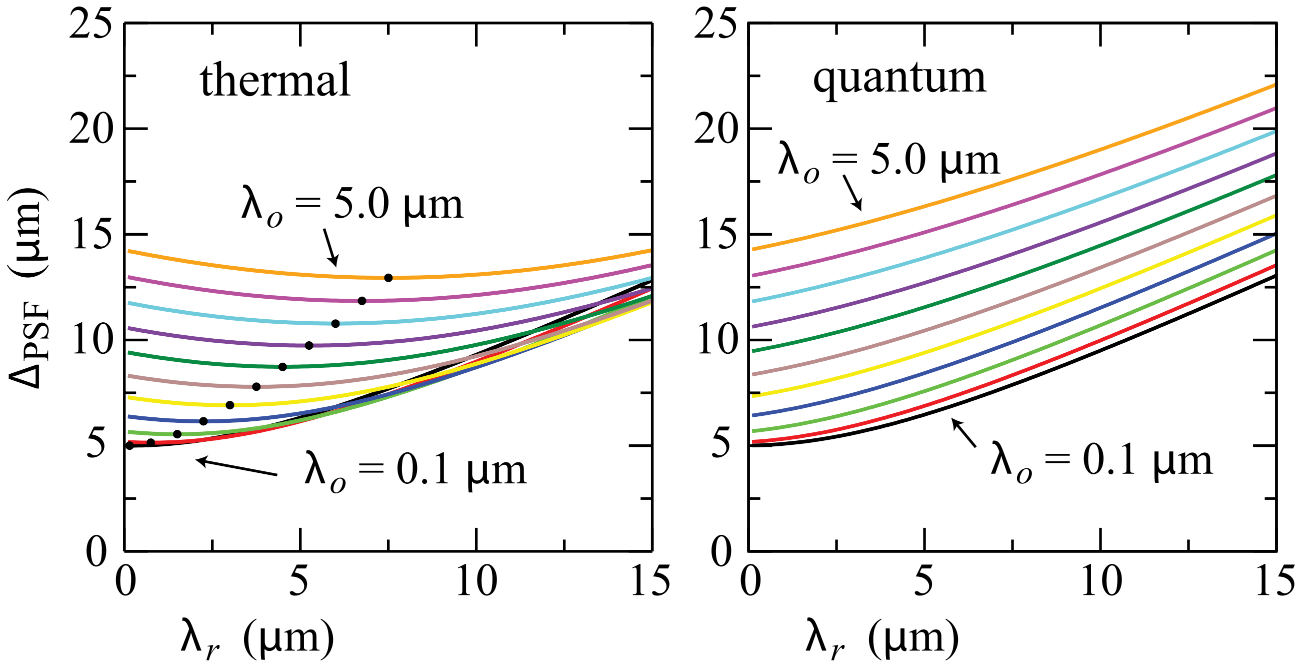
Statement of Research Results

We have had a very productive year in terms of research productivity. In the area of *Ghost Imaging*, we completed one study [B7] that describes quantitatively how the properties of ghost imaging are modified when different colors of light are used in the object and reference arms. We also completed a study [B17] that shows how the quality of the ghost image can be improved by using higher-order correlations of the intensities of the object and reference beams. We also have new results in the area of *Single-Photon Imaging*. We published one study [B8] that shows that by means of a holographic method we can discriminate between two objects even when they are illuminated by only a single photon. In a related study [B12] we showed that we can discriminate among four objects using a single biphoton in a ghost imaging configuration. We have also studied [B6, B9, B11] the properties of light fields with transverse distributions that impart an *Orbital Angular Momentum (OAM)* onto the photon. These OAM states constitute a complete basis, and thus any quantum image can be described in terms of these states. Our work has quantified the thought that these states can be used as carriers of quantum information. We have also obtained new results in the area of *Quantum Technologies*. One of these studies [B2] shows how Bayesian statistics can be used to provide a better estimate of the number of photons contained in a light field. Another study [B10] provides a laboratory demonstration of a new method that can be used to increase the nonlinear optical response of materials useful in multiphoton detection. In addition, we have completed studies [B1, B3, B4, B5] of *Fundamental Properties of Quantum Light Fields*.

Conclusions

- There is a microscopic cascaded contribution to $\chi^{(5)}$ induced purely by local-field effects.
- This contribution can be larger than the macroscopic cascaded term.
- Microscopic cascading can induce high-order nonlinearities useful for quantum information.

Two-Color Ghost Imaging: Results



$\sigma_x = 5\,\mu\text{m}$
$w = 10\,\text{mm}$
$D = 20\,\text{mm}$
$l_{r1} = 100\,\text{mm}$
$l_o = 150\,\text{mm}$

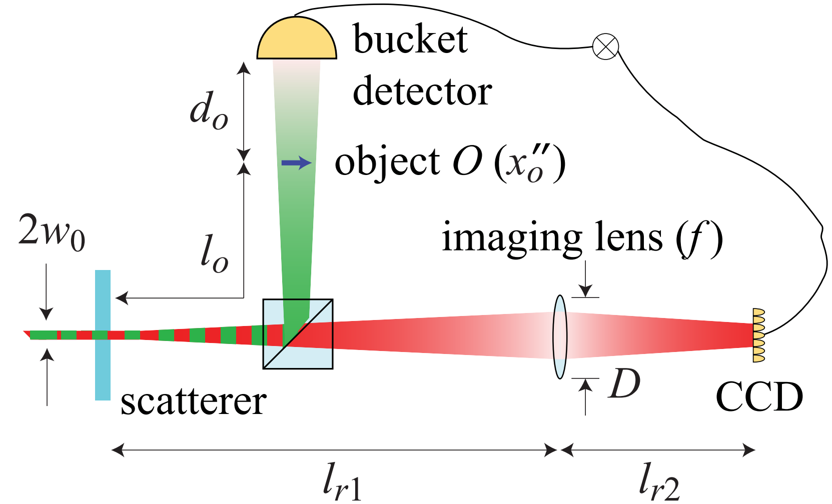
thermal:

$$\Delta_{\text{PSF}} = \sqrt{\sigma_x^2 + \left(\frac{\lambda_o l_o}{2\pi w}\right)^2 + \frac{(\lambda_r l_{r1} - \lambda_o l_o)^2}{(\lambda_r l_{r1}/w)^2 + (2\pi D)^2}}.$$

quantum:

$$\Delta_{\text{PSF}} = \sqrt{\sigma_x^2 + \left(\frac{\lambda_o l_o}{2\pi w}\right)^2 + \frac{(\lambda_r l_{r1} + \lambda_o l_o)^2}{(\lambda_r l_{r1}/w)^2 + (2\pi D)^2}}.$$

In many practical situations, this term dominates.



Chan, O'Sullivan, Boyd, PRA 2009